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В сборник включены тезисы докладов, представленных на XXXII Международный семинар по проблемам устойчивости стохастических моделей (ISSPSM'2014) и VI Международный рабочий семинар "Прикладные задачи теории вероятностей и математической статистики, связанные с моделированием информационных систем" (APTP + MS'2014) (летняя сессия).

On special empirical processes of independence

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Consider a following model of experiments in which observed pairs are consists of $\{(X_k, A_k), k \geqslant 1\}$, where X_k are random elements defined on a probability space $(\Omega, \mathcal{A}, \mathbb{P})$ with values in a measurable space $(\mathfrak{X}, \mathfrak{B})$. Events A_k have a common probability $p \in (0,1)$. Let $\delta_k = I(A_k)$ is indicator of the event A_k. At the $n-$ th step of experiments is observed a sample $\mathbb{S}^{(n)} = \{(X_k,\, \delta_k),\ 1 \leqslant k \leqslant n\}$. Each pair in the sample $\mathbb{S}^{(n)}$ induced a statistical model with sample space $\mathfrak{X} \otimes \{0,1\}$, σ-algebra of sets of the form $B \times D$ and induced distribution $\mathbb{Q}^*(B \times D) = \mathbb{P}(X_k \in B, \delta_k \in D)$, where $B \in \mathfrak{B}$, $D \subset \{0,1\}$. Define submeasures $\mathbb{Q}_1(B) = \mathbb{Q}^*(B \times \{1\})$, $\mathbb{Q}_0(B) = \mathbb{Q}^*(B \times \{0\})$ and $\mathbb{Q}(B) = \mathbb{Q}^*(B \times \{0,1\}) = \mathbb{Q}_0(B) + \mathbb{Q}_1(B)$, $B \in \mathfrak{B}$ and its estimates $\mathbb{Q}_{1n}(B) = \frac{1}{n}\sum\limits_{k=1}^{n} \delta_k I(X_k \in B)$, $\mathbb{Q}_{0n}(B) = \frac{1}{n}\sum\limits_{k=1}^{n}(1-\delta_k) I(X_k \in B)$, $\mathbb{Q}_n(B) = \frac{1}{n}\sum\limits_{k=1}^{n} I(X_k \in B) = \mathbb{Q}_{0n}(B) + \mathbb{Q}_{1n}(B)$ for all $B \in \mathfrak{B}$. Consider the hypothesis $\mathcal{H}$ of independence of X_k and A_k for each $k \geqslant 1$.

We consider general classes of specially normalized empirical processes of independence indexed by the class $\mathcal{F}$ of measurable functions $f : \mathfrak{X} \to \mathbb{R}$. For a signed measure $\mathbb{G}$ and function $f \in \mathcal{F}$ denote the integral

$$\mathbb{G}f = \int\limits_{\mathfrak{X}} f d\mathbb{G}.$$

Define $\mathcal{F}$ - indexed empirical process $\mathbb{G}_n : \mathcal{F} \in \mathbb{R}$ as:

$$f \mapsto \mathbb{G}_n f = \sqrt{n}\left(\mathbb{Q}_n - \mathbb{Q}\right) f = n^{-1/2} \sum_{k=1}^{n} \left(f(X_k) - \mathbb{Q}f\right),\ f \in \mathcal{F}.$$

Note that $\mathbb{G}_n f = \mathbb{G}_{0n} f + \mathbb{G}_{1n} f$, where $\mathbb{G}_{jn} f = \sqrt{n}\left(\mathbb{Q}_{jn} - \mathbb{Q}_j\right) f, j = 0, 1$. Donsker-type theorems provide a general conditions on $\mathcal{F}$, under which

$$\mathbb{G}_n f \Rightarrow \mathbb{G} f \ \text{ in } \ l^{\infty}(\mathcal{F}), \tag{1}$$

where $l^{\infty}(\mathcal{F})$ - the space of all bounded functions $f : \mathfrak{X} \to \mathbb{R}$ equipped with the supremum - norm $\|f\|_{\mathcal{F}}$ and $\Rightarrow$ means the weak convergence. Limiting field $\{\mathbb{G}f,\ f \in \mathcal{F}\}$ called $\mathbb{Q}$ - Brownian bridge. In connection with the problem of testing the hypothesis $\mathcal{H}$, we introduce $\mathcal{F}$ - processes $\Lambda f = \mathbb{Q}_1 f - p\mathbb{Q}f$, $\Lambda_n f = \mathbb{Q}_{1n} f - p_n \mathbb{Q}_n f$, $f \in \mathcal{F}$ and

$$\Delta_n f = \int\limits_{\mathfrak{X}} f d\Delta_n = \left(\frac{n}{p_n\left(1 - p_n\right)}\right)^{1/2} \left(\Lambda_n - \Lambda\right) f,\ f \in \mathcal{F}. \tag{2}$$

One of important properties of the process (2) is its convergence to the same $\mathbb{Q}$-Brownian bridge $\{\mathbb{G}f\ ,\ f\in\mathcal{F}\}$ under validity of $\mathcal{H}$.

To present the basic theorems we define the complexity or entropy of class $\mathcal{F}$. Bracketing (or covering) number $N_{[\,]}\left(\varepsilon,\mathcal{F},\mathcal{L}_q\left(\mathbb{Q}\right)\right)$ is the minimum number of ε - brackets in $\mathcal{L}_q(\mathbb{Q})$ needed to cover $\mathcal{F}$ (see Shorack and Wellner [1], Van der Vaart and Wellner [2]):

$$N_{[\,]}\left(\varepsilon,\mathcal{F},\mathcal{L}_q\left(\mathbb{Q}\right)\right)=\min\left\{\begin{array}{l} k:\ \text{for some } f_1,...,f_k\in\mathcal{L}_q\left(\mathbb{Q}\right),\\ \mathcal{F}\subset\bigcup\limits_{i,j}\left[f_i,f_j\right]:\left\|f_j-f_i\right\|_{\mathbb{Q},q}\leqslant\varepsilon.\end{array}\right.$$

For weak convergence of $\mathcal{F}$ - indexed empirical processes (2) we need the integral of the metric entropy with bracketing to be

$$J_{j[\,]}^{(q)}\left(\delta\right)=J_{j[\,]}\left(\delta;\mathcal{F};\mathcal{L}_q\left(\mathbb{Q}_j\right)\right)=\int\limits_0^{\delta}\left(H_{jq}\left(\varepsilon\right)\right)^{1/2}d\varepsilon,j=0,1,\ \text{for } 0<\delta<1,$$

where $H_{jq}\left(\varepsilon\right)=\log N_{[\,]}\left(\varepsilon,\mathcal{F},\mathcal{L}_q\left(\mathbb{Q}_j\right)\right)$ is metric entropy of class $\mathcal{F}$ in $\mathcal{L}_q(\mathbb{Q}_j)$, $j=0,1$. We introduce the following conditions:

(i) Let the class $\mathcal{F}$ such that

$$\mathcal{F}\subset\mathcal{L}_2(\mathbb{Q}_j)\ \text{and}\ J_{j[\,]}^{(2)}\left(1\right)<\infty,\ j=0,1. \tag{3}$$

Theorem 1. *Under the conditions (3) for* $n\to\infty$

$$\Delta_n f\Rightarrow\Delta f\ \ in\ l^{\infty}(\mathcal{F}), \tag{4}$$

where $\{\Delta f,\ f\in\mathcal{F}\}$ *is a Gaussian field with zero mean and under validity of the hypothesis* $\mathcal{H}$*, it coincides in distribution with* $\mathbb{Q}$ *- Brownian bridge.*

Now concider case of random sample size. Let the sequence N_n of Poisson r.v.-s with mean n. Suppose that sequences $\{N_n,\ n\geq 1\}$ and $\{(X_k,\delta_k),\ k\geq 1\}$ are independent. Let $\left\{\Delta^*_{N_n}f,\ f\in\mathcal{F}\right\}$ be sequence of normalized empirical processes of independence obtained from (2) by replacing upper index n of all summation to a random sequence N_n .

Theorem 2. *Under the conditions (3) at* $n\to\infty$

$$\Delta^*_{N_n}f\Rightarrow\Delta^*f\ \ in\ l^{\infty}(\mathcal{F}), \tag{5}$$

where by hypothesis $\mathcal{H}$*,* $\Delta^*f\overset{d}{=}\mathbb{W}(f)$*,* $f\in\mathcal{F}$*. Here* $\{\mathbb{W}(f),f\in\mathcal{F}\}$ *is Brownian sheet.*

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High level subcritical branching processes in a random environment

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Let $\{\xi_n,\ n \in \mathbb{N}_0\}$ be a *branching process in a random environment* (BPRE) defined by a sequence of independent and identically distributed (random) generating functions $\{f_n(s),\ n \in \mathbb{N}\}$. Note that ξ_n is the size of the nth generation (we assume that $\xi_0 = 1$). The generating function $f_n(s)$, $s \in [0,1]$, defines the reproduction law for the particles in the $(n-1)$th generation, $n \in \mathbb{N}$.

Assuming that $f_1'(1) \in (0, +\infty)$ a.s., we set $X_i = \ln f_i'(1)$ for $i \in \mathbb{N}$. Note that the random variables $X_1, X_2, \ldots$ are independent and identically distributed. Introduce the *associated random walk* $S_0 = 0$, $S_n = \sum_{i=1}^n X_i$, $n \in \mathbb{N}$.

Suppose that the process $\{\xi_n\}$ is *subcritical*, i.e. $\mathbf{E}X_1 < 0$, and there exists a positive number $\varkappa$ such that

$$\mathbf{E}\exp(\varkappa X_1) = 1, \qquad \mathbf{E}(|X_1|\exp(\varkappa X_1)) < +\infty. \tag{1}$$

Condition (1) is classical for random walks with negative drift and allows one to pass to *conjugate random walk* with positive drift. In addition, we assume that

$$\mathbf{E}(\xi_1 \ln(\xi_1 + 1)\exp((\varkappa - 1)X_1)) < +\infty, \tag{2}$$

and if $\varkappa \geq 1$, then there exists a number $p > \varkappa$ such that

$$\mathbf{E}(\xi_1^p \exp((\varkappa - p)X_1)) < +\infty. \tag{3}$$

Introduce the *first passage time* of the process $\{\xi_n\}$ to a level $x > 1$:

$$T_x = \min\{n : \xi_n > x\},$$

and the *lifetime* of the process $\{\xi_n\}$:

$$T = \min\{n : \xi_n = 0\}.$$

In [1] and [2], the author showed that if conditions (1)-(3) are satisfied, then

$$\mathbf{P}(T_x < +\infty) \sim c_0 x^{-\varkappa},$$

$$\left\{ \frac{T_x}{\ln x} \,\middle|\, T_x < +\infty \right\} \xrightarrow{\mathbf{P}} \frac{1}{a},$$

$$\left\{ \frac{T}{\ln x} \;\middle|\; T_x < +\infty \right\} \xrightarrow{\mathbf{P}} \frac{1}{a} - \frac{1}{b}$$

as $x \to +\infty$, where c_0 is a positive constant, $a = \mathbf{E}\left(X_1 \exp\left(\varkappa X_1\right)\right)$, $b = \mathbf{E}X_1$.

In addition, we assume that

$$\mathbf{E}\left(X_1^2 \exp\left(\varkappa X_1\right)\right) < +\infty. \tag{4}$$

Set $\sigma^2 = \mathbf{E}\left(X_1^2 \exp\left(\varkappa X_1\right)\right) - a^2$. Let $B = \{B\left(t\right),\ t \in [0,1]\}$ be a standard Brownian motion. The following *functional limit theorem* for the first passage time to different levels is valid.

Theorem 1. *If* $\{\xi_n,\ n \in \mathbb{N}_0\}$ *is a subcritical BPRE and conditions* (1)-(4) *hold, then*

$$\left\{ \frac{T_{x^t} - t\ln x/a}{\sigma a^{-3/2}\sqrt{\ln x}},\ t \in [0,1] \;\middle|\; T_x < +\infty \right\} \xrightarrow{D} B$$

as $x \to +\infty$, *where the symbol* $\xrightarrow{D}$ *means convergence in distribution in the space* $D\left[0,1\right]$ *with the Skorokhod topology.*

Also the following *functional limit theorem* for the size of different generations is valid.

Theorem 2. *If* $\{\xi_n,\ n \in \mathbb{N}_0\}$ *is a subcritical BPRE and conditions* (1)-(4) *hold, then*

$$\left\{ \frac{\ln \xi_{\lfloor ty/a \rfloor} - ty}{\sigma\sqrt{y/a}},\ t \in [0,1) \;\middle|\; T_{\exp y} < +\infty \right\} \xrightarrow{D} B$$

as $y \to +\infty$.

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Statistical analysis of queueing system with regenerative input

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We focus on statistical estimations of parameters of queueing systems with regenerative input flow. One can find the definition of regenerative flow in [1]. Regenerative flow represents natural generalization of many kinds of flows considering in the queueing theory. Besides, a regenerative flow has some useful properties that make it possible to investigate various applied models. Unfortunately, for queueing systems with rather complicated input flows it is impossible (with rare exceptions) to obtain explicit expressions of their operating characteristics such as average queue length, average waiting time and so on. Therefore the proof of the theorems concerning stability conditions ang heavy traffic situation becomes important. There is a whole series of such results obtained in resent years. We cite as an example a theorem related to a single-server queueing system. This theorem were proved in [1] We use the following notation

- τ_i is the length of the ith regeneration period; $\mu = \mathsf{E}\tau_i$, $\sigma_\tau^2 = \mathsf{Var}\tau_i$
- ξ_i is the number of customers entering the system during the ith regeneration period; $a = \mathsf{E}\xi_i$, $\sigma_\xi^2 = \mathsf{Var}\xi_i$
- $\lambda = a/\mu$; $r_{\xi,\tau} = \mathsf{cov}(\xi,\tau)$
- η_i is the service time of the ith customer; $b = \mathsf{E}\eta_i$, $\sigma_\beta^2 = \mathsf{Var}\eta_i$
- $N(u)$ - renewal process generated by the sequence $\{\tau_i\}_{i=1}^\infty$,
- $q(t)$ and $W(t)$ are the processes of queue length and waiting time respectively

Theorem 1. *Let $\{X(u)\, u \geqslant 0\}$ be a regenerative flow and $\mathsf{E}\xi_1^{2+\delta} < \infty$, $\mathsf{E}\tau_1^{2+\delta} < \infty$ for some $\delta > 0$. Then the normalized processes $q_T(tT)/\sqrt{T}$ and $W_T(tT)/\sqrt{T}$ C-converge on any finite interval [0; h] to diffusion processes with reflecting zero boundary and coefficients $(b^{-1}, b^2\sigma_W^2) and (1, \sigma_W^2), respectively. Here$*

$$\sigma_W^2 = \frac{\sigma_\beta^2}{b} + \frac{b}{\lambda}\sigma_x^2$$

and

$$\sigma_x^2 = \frac{\sigma_\xi^2}{\mu} + \frac{a^2\sigma_\tau^2}{\mu^3} - \frac{2ar_{\xi,\tau}}{\mu^2}$$

For practical applications of this theorem and analogous results for more complicated systems obtained, e.g., in [1,2] it is necessary to estimate the parameters σ_x^2 and λ. The consistent estimate of intensity λ is of the form

$$\widehat{\lambda}(t) = \frac{X(t)}{t}.$$

Now we would like to estimate the coeffitient σ_x^2. If the both processes $\{X(u), N(u),\, u \in (0,t]\}$ are observable one can apply classical methods for estimation the parameters a, μ, σ_ξ^2, σ_τ^2, $r_{\xi,\tau}$. Problem arise when only process $X(t)$ is observable. Thus it is necessary to use another approach. Choosing some $A > 0$ we denote

$$Z_n(A) = X(kA) - X((k-1)A), \quad k = 1, 2, \ldots,$$

$$\widehat{\Delta}_n = \frac{1}{nA} \sum_{k=1}^{n} (Z_k(A) - \widehat{\lambda}(nA)A)^2.$$

Theorem 2. *Let $\{X(u)\, u \geqslant 0\}$ be a regenerative flow and* $\mathsf{E}\xi_1^{2+\delta} < \infty$, $\mathsf{E}\tau_1^{2+\delta} < \infty$ *for some $\delta > 0$. Then $\widehat{\Delta}_n$ is consistent estimate for coefficient σ_x^2 as $A \to \infty$ and $n \to \infty$.*

Further we discuss different ratios between n and A and consider some examples.

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Limit theorems for queuing system with an infinite number of servers

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This article focuses on the system with an infinite number of servers. Arriving customers form a doubly stochastic Poisson process(DSPP) $A(t)$, which is defined as follows [2]:

$$A(t) = A^*(\Lambda(t))$$

where $\{A^*(t), t \geq 0\}-$ is a standard Poisson process, and $\{\Lambda(t), t \geq 0\}-$ is a stochastic process with non-decreasing right-continuous trajectories not depending on $A^*(t)$, $\Lambda(0) = 0$.

Condition 1. *The process $\Lambda(t)$ has the following form $\Lambda(t) = \int\limits_0^t \lambda(y, \omega)dy$, where $\lambda(y)-$ is a non-negative bounded stationary stochastic process such that*

$$|r(x)| = |cov(\lambda(0), \lambda(x))| \leq \begin{cases} c_0 & \text{for } 0 < x < a, \\ c_0 x^{-\alpha} & \text{for } x \geq a. \end{cases} \tag{1}$$

Here $\alpha > 0$ and c_0, $a-$ are certain positive constants.
We denote $E\lambda(t) = \lambda$.
The process $\Lambda(t)$ is called the leading process and $\lambda(t)$ is the intensity of doubly stochastic Poisson process $\{A(t), t \geq 0)\}$.

The service times of customers form a sequence $\{\eta_i\}_{i=1}^{\infty}$ of independent identically distributed random variables with a distribution function $B(x)$. We denote $\overline{B}(x) = 1 - B(x)$.

Condition 2. *For some positive constants c_1, c_2, t_0*

$$c_1 t^{-\Delta} \leq \overline{B}(x) \leq c_2 t^{-\Delta},\ 0 < \Delta < 1, \tag{2}$$

for all $t \geq t_0$.

It follows from (2) that $\int_0^\infty x dB(x) = \infty$.

Let $q(t)$ be the number of customers in the system at time t. We would like to study asymptotic behavior of the process $q(t)$ as $t \to \infty$. The analogous problem was considered in [1] for a system $GI/GI/\infty$.

The main focus of this paper is to examine the process $q(t)$, which is the number of customers in the system at time t.

Using the properties of the DSPP [2], we obtain the formula for the probability distribution of $q(t)$.

$$P(q(t) = k) = E\left(e^{-\rho(t)} \frac{(\rho(t))^k}{k!}\right), \tag{3}$$

where $\rho(t) = \int_0^t \overline{B}(t-x)\lambda(x)dx$.

Conditions 1 and 2 allow us to find estimates for $E\rho(t)$ and $Var\rho(t)$. For the first moment we get

$$\lambda c_1 t^{1-\Delta} \le E\rho(t) \le \lambda c_2 t^{1-\Delta}. \tag{4}$$

The estimation for $Var\rho(t)$ follows from the next lemma.

Lemma 1 *Suppose that conditions 1 and 2 are fulfilled. Then for any* $0 < \gamma < 1$, $\delta > 0$ *there exists a positive constant* C*, such that for sufficiently large* t *the following inequality holds*

$$\frac{Var\rho(t)}{C} \le t^{\gamma+\delta} + t^{1+\gamma-\alpha}\ln t + t^{\delta(\alpha-1)+\gamma}\ln t + t^{\delta(\alpha-1)+1-2\Delta}\ln t. \tag{5}$$

With these estimations it becomes possible to prove the following limit theorems. Let $\beta(t) = \int_0^t \overline{B}(x)dx$, so that $E\rho(t) = \lambda\beta(t)$.

Theorem 1 *If* $\alpha > \Delta$ *and conditions 1,2 are fulfilled then for any fixed* t

$$\frac{q(t) - \lambda\beta(t)}{\sqrt{\lambda\beta(t)}} \xrightarrow{d} \mathcal{N}(0,1),$$

as $t \to \infty$.

Theorem 2 *If* $\alpha > 2\Delta - 1$ *and conditions 1,2 are fulfilled, then*

$$\frac{q(t)}{\lambda\beta(t)} \xrightarrow{p} 1,$$

as $t \to \infty$.

Also we give two corollaries of these theorems.

Corollary 1 *Let* $\lambda(t)$ *be a stationary bounded regenerative process, with* $\overline{F}(t) \le ct^{-\alpha-1}$ *as a distribution function of regeneration period. Then*

- *Theorem 1 holds for* $\alpha > \Delta$,
- *Theorem 2 holds for* $\alpha > 2\Delta - 1$.

Corollary 2 *Let the input flow* $A(t)$ *be a Markov-modulated process, and its control Markov chain be a birth and death process with* $\lambda_j = j\lambda$, $\mu_j = j\mu$, $j \ge 0$, $\mu > \lambda$. *Then for any* $\alpha > 0$ *and* $0 < \Delta < 1$ *Theorems 1 and 2 are fulfilled.*

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Asymptotic analysis of a heterogeneous multi-server system with renewal-type service interruptions

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This study is focused on a multi-server queueing system with a regenerative input flow. We assume that every server may has distinct service time distribution and they are not always available for operations. Servers' interruptions may result from different reasons, such as resource sharing, servers breakdowns and repairs, and servers vacations. Systems with unreliable servers have been intensively investigated for a long time. The main point was focused on the single-server case. The framework of problems and their solutions are presented in (Krishnamoorthy et al., 2012). In this study some generalizations of queueing models with service interruptions are investigated. Firstly, the input flow is assumed to be regenerative. The class of regenerative processes contains most of fundamental flows that are exploited in queueing theory including recurrent, semi-Markov, Markov-modulated, Markov arrival process and others (see, e.g. (Afanasyeva, Bashtova, 2014)). Secondly, the breakdowns of the servers may occur at any time even if they are not occupied by customers. Consecutive moments of breakdowns are defined by a renewal process. We consider the preemptive resume service discipline (discipline D_1) as well as the preemptive repeat different service discipline (discipline D_2). In the former case, the service continues after interruption whereas service is repeated from the beginning with different independent service time in the latter case. For the models the necessary and sufficient conditions of stability and functional limit theorems are established. The key element of our analysis is the coupling of processes under consideration. This method is based on the strong regeneration property of the input flow and renewal structure of processes describing the servers' breakdowns (Afanasyeva, Bashtova, 2014). We also employ very effective approach based on the constructions of so-called autonomous system (Whitt, 2002).

Now we describe our models. Let $X(t)$ be a regenerative input flow to the system S^d $(d = 1, 2)$ with intensity $\lambda = \lim_{t\to\infty} \frac{X(t)}{t}$, where $d = 1$ if the service discipline is D_1 and $d = 2$ if it is D_2. The system S^d has m heterogeneous servers. By $B_i(t)$, $i = \overline{1, m}$ denote a distribution function of service times $\{\eta_n^i\}_{n=1}^{\infty}$ by the ith server, b_i is its' mean, and $H_i(t)$ is a renewal process defined by $\{\eta_n\}_{n=1}^{\infty}$. Suppose that working periods of the ith server $\{u_{in}^{(1)}\}_{n=1}^{\infty}$ are random variables with mean $a_i^{(1)}$ and $u_{in}^{(1)} = v_{in}^{(1)} + v_{in}^{(2)}$, where $v_{in}^{(1)}$ has exponential distribution. Periods of the ith server reconstructions $\{u_{in}^{(2)}\}_{n=1}^{\infty}$ are random variables with mean $a_2^{(1)}$. By $Q^d(t)$ $(d = 1, 2)$ denote the number of customers in the system S^d. Let us formulate main results that hold under some not restrictive conditions.

Theorem 1. *The process $Q^d(t)$ is ergodic iff $\rho_d < 1$, $(d = 1, 2)$, where*

$$\rho_1 = \frac{\lambda}{\mu_1}, \quad \mu_1 = \sum_{i=1}^{m} \frac{a_i^{(1)}}{(a_i^{(1)} + a_i^{(2)})b_i}, \quad \text{for discipline } D_1,$$

$$\rho_2 = \frac{\lambda}{\mu_2}, \quad \mu_2 = \sum_{i=1}^{m} \frac{EH_i(u_i^{(1)})}{(a_i^{(1)} + a_i^{(2)})}, \quad \text{for discipline } D_2.$$

For functional limit theorems we introduce the following scaled processes

$$Q_n^d(t) = \frac{Q^d(nt) - (\lambda - \mu_d)nt}{\sigma_{Q^d}\sqrt{n}},$$

where σ_{Q^d} is some constant.

Theorem 2. *If $\rho_d > 1$ $(\rho_d = 1)$, then the process $Q_n^d(t)$ weakly converges to a standard Brownian motion (absolute value of a standard Brownian motion) on any finite interval $[0, v]$ as $n \to \infty$.*

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Ergodic theorem for a single-server queue in a random environment

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We consider one-channel queue with an unreliable server. The input $A(t)$ is supposed to be a regenerative flow with points of regeneration $\{\theta_j\}_{j=1}^{\infty}, \theta_0 = 0$. The definition and properties of this flow can be found in [2].

The service times are defined by the sequence $\{\eta_j\}_{j=1}^{\infty}$ of i.i.d.r.v.'s with d.f. $B(x)$ and finite moment $b = \mathsf{E}\eta_j$. Besides, sequence $\{\eta_j\}_{j=1}^{\infty}$ does not depend on $A(t)$.

Let $X(t)$ be the total service time of customers arriving at the system during time interval $[0,t)$, i.e. $X(t) = \sum_{j=1}^{A(t)} \eta_j$. Then $X(t)$ is also regenerative flow with the same points of regeneration as process $A(t)$.

The server can be failed, its breaks and interval between recoveries depend on a stochastic process $U(t)$ that is an ergodic Marcov chain not depending on $X(t)$ with set of states $\mathbb{E} = (0, 1, \dots)$.

When process $U(t)$ achieves the state $i (i \in \mathbb{E})$ the working server fails with probability $\alpha_i \geq 0$ and broken server recovers with probability $\beta_i \geq 0$.

It is assumed that there are states of process $U(t)$ - i_0 and i_1, such that $\alpha_{i_0} > 0, \beta_{i_1} > 0$.

Let $W(t)$ be the workload process. Then the following relation takes place

$$W(t) = \sup_{0 \leq u \leq t} (W(0) + Z(t), Z(t) - Z(u))$$

where $Z(t) = X(t) - Y(t)$, and $Y(t) = \int_0^t e(s)ds$, where $e(s) = 1$ if server in the working state at moment t and $e(t) = 0$ otherwise. It means that stochastic process $N(t) = \{e(t), U(t)\}$ is a random environment for $W(t)$ (see for example [1]).

We note that $N(t)$ is ergodic Markov chain and $\pi = \lim_{t\to\infty} \mathsf{P}(e(t) = 1)$. The coefficient traffic of the system is given by the $\rho = \frac{\lambda b}{\pi}$.

Theorem 1. *If* $\rho \geq 1$ *then* $W(t) \xrightarrow[t\to\infty]{\mathsf{P}} \infty$ *and if* $\rho < 1$ *then* $\lim_{t\to\infty} \mathsf{P}(W(t) \leq x) = F(x)$ *exists and* $F(x)$ *is nonsingular d.f.*

The proof is based on results from [1] for cyclic queues. Some examples are also give.

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Portfolio analysis with transaction costs under uncertainty

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In [1] we investigated portfolios with transaction costs for one-period deterministic model and derived the portfolio price return formula. As it mentioned in [1] unlike one-period transaction with a single asset, the scheme for calculation the real net return on a one-period portfolio transaction has a number of features. The aim of this work is to investigate analogous problems under uncertainty.

Let us introduce some notation. Suppose that we have n assets $A_1, \dots, A_n$. Let R_k ($r_k = E(R_k)$ resp.) denotes the random price return (expected price return resp.) on A_k. The portfolio will be denoted by the vector of asset weights $x^T = (x_1, \dots, x_n)$: $\sum\limits_{k=1}^{n} x_k = 1$. If there are no commission costs, it is well known [2–5] that the portfolio return $R(x)$ (expected return $r(x)$ resp.) is the weighted average of the individual asset returns (expected returns resp.):

$$R(x) = R_1 x_1 + \dots, R_n x_n, \qquad r(x) = r_1 x_1 + \dots, r_n x_n.$$

It follows that the portfolio variance of return or risk is given by $V(x) = x^T C x$, where C is the covariance matrix of asset returns. In the future, we need the concept of the investor's utility function $U(x)$, which is defined as a linear function of the mean and variance of the portfolio: $U(x) = r(x) - \frac{\theta}{2} V(x)$, where θ is the investor's risk tolerance.

Now consider the portfolio x with commission α. Then the random portfolio price return $R_\alpha(x)$ and the expected portfolio price return $r_\alpha(x)$ are defined respectively by the formulas

$$R_\alpha(x) = \frac{\sum\limits_{k=1}^{n}(x_k - \alpha|x_k|)R_k - 2\alpha||x||}{1+\alpha||x||} = \frac{R(x) - \alpha\sum\limits_{k=1}^{n}(2+R_k)|x_k|}{1+\alpha||x||}, \qquad (1)$$

$$r_\alpha(x) = \frac{\sum\limits_{k=1}^{n}(x_k - \alpha|x_k|)r_k - 2\alpha||x||}{1+\alpha||x||} = \frac{r(x) - \alpha\sum\limits_{k=1}^{n}(2+r_k)|x_k|}{1+\alpha||x||}. \qquad (2)$$

Here $||x|| = \sum\limits_{k=1}^{n}|x_k|$. Note that $R_0(x) = R(x)$ and $r_0(x) = r(x)$. From (1) it follows that the portfolio variance of return or risk is given by

$$V_\alpha(x) = \frac{\tilde{x}^T C\tilde{x}}{(1+\alpha||x||)^2} = \frac{1}{(1+\alpha||x||)^2}(V(x) - 2\alpha|x|^T Cx + \alpha^2|x|^T C|x|), \quad (3)$$

where $\tilde{x}^T = (x_1 - \alpha|x_1|, \ldots, x_n - \alpha|x_n|)$, $|x|^T = (|x_1|, \ldots, |x_n|)$. Note that $V_0(x) = V(x)$.

Define the the portfolio utility function by $U_\alpha(x) = r_\alpha(x) - \frac{\theta}{2}V_\alpha(x)$.

In what follows we distinguish between two portfolio types (models). The first model we call the Black's model. For this model it is assumed only the budget constraint $x_1 + \ldots + x_n = 1$.The second model we call the Markowitz model. It differs from Black's model by the additional constraints $x_i \geqslant 0$ (short positions are prohibited). It is significant that unlike the ideal case when $\alpha = 0$ the task of choosing the optimal portfolio for Black's model is unsmooth. Furthermore, it may happen that the minimal variance portfolio and the portfolio with maximum utility without commission have positive returns, but these portfolios with the commission have negative returns.

Formulas (1) and (2) can be simplified for the Markowitz model. Namely

$$r_\alpha(x) = a_\alpha r(x) - \beta_\alpha, \qquad V_\alpha(x) = a_\alpha V(x), \qquad U_\alpha(x) = a_\alpha U(x) - \beta_\alpha.$$

where $a_\alpha = \frac{1-\alpha}{1+\alpha}$, $\beta_\alpha = \frac{2\alpha}{1+\alpha}$. Moreover, we have

$$a_\alpha \underline{r} - \beta_\alpha \leqslant r_\alpha(x) \leqslant a_\alpha \overline{r} - \beta_\alpha,$$

where $\underline{r} = \min\{r_1, \ldots, r_n\}$, $\overline{r} = \max\{r_1, \ldots, r_n\}$. According to these formulas, the minimal variance portfolio and the portfolio with maximum utility don't dependent on the commission.

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Parameter estimation for subcritical Heston models based on discrete time observations

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Heston models have been extensively used in financial mathematics since one can well-fit them to real financial data set, and they are well-tractable from the point of view of computability as well. Hence parameter estimation for Heston models is an important task.

In the talk we study conditional least squares estimators (CLSEs) and least squares estimators (LSEs) for Heston models

$$\begin{cases} dY_t = (a - bY_t)\,dt + \sigma_1\sqrt{Y_t}\,dW_t, \\ dX_t = (\alpha - \beta Y_t)\,dt + \sigma_2\sqrt{Y_t}\big(\varrho\,dW_t + \sqrt{1-\varrho^2}\,dB_t\big), \end{cases} \qquad t \geq 0, \qquad (1)$$

where $a > 0$, $b, \alpha, \beta \in \mathbb{R}$, $\sigma_1 > 0$, $\sigma_2 > 0$, $\varrho \in (-1, 1)$, and $(W_t, B_t)_{t\geq 0}$ is a 2-dimensional standard Wiener process. We investigate only the so-called subcritical case, i.e., when $b > 0$. It is well-known that in this case the process $(Y_t)_{t\geq 0}$, which is just the Cox–Ingersoll–Ross process, is ergodic. We consider a CLSE and LSE of (a, b, α, β) based on discrete time observations of the process $(X_t, Y_t)_{t\geq 0}$, when the parameters σ_1, σ_2 and ϱ are assumed to be known.

We use the method of conditional least squares, which was first applied to the CIR process by Overbeck and Rydén [1]. We estimate a suitably transformed parameter vector (c, d, γ, δ), for which the estimation error can be written as a sum of martingale differences. The strong consistency and the asymptotic normality follow from this fact using the strong law of large numbers and the central limit theorem for square-integrable martingales. The asymptotic covariance matrix is derived for the estimation errors of (c, d, γ, δ) as well as the estimation errors of the original parameters (a, b, α, β).

We also introduce a plausible set of estimators based on the ordinary least squares method, show that they are not consistent, and we derive their strong limit.

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Statistical inference of continuous state and continuous time branching processes with immigration

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First a Feller type diffusion approximation is derived for random step functions formed from a critical, positively regular multi-type continuous state and continuous time branching processes with immigration (CBI processes). Based on this result, the asymptotic behavior of the conditional least squares estimators of the offspring means for a 2-type critical doubly symmetric positively regular CBI process is described.

In the proofs, moment formulas and moment estimations play a crucial role, which are based on an identification of a multi-type CBI process as a pathwise unique strong solution of certain stochastic differential equation with jumps, see Barczy at. al [2], where a generalization of Yamada-Watanabe results for stochastic differential equations with jumps is used, see Barczy at. al [1].

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Martin kernel for fractional Laplacian in narrow cones

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For $d \geq 2$ and $0 < \Theta < \pi$, we consider the right circular cone of angle Θ:

$$\Gamma_\Theta = \left\{ x = (x_1, \ldots, x_d) \in \mathbb{R}^d : \ x_d > |x| \cos \Theta \right\}.$$

The Martin kernel of the fractional Laplacian $\Delta^{\alpha/2}$, $0 < \alpha < 2$, for Γ_Θ is the unique continuous function $M \geq 0$ on $\mathbb{R}^d$, such that M is smooth on Γ_Θ, $\Delta^{\alpha/2} M = 0$ on Γ_Θ, $M = 0$ on Γ_Θ^c, and $M(1, 0, \ldots, 0) = 1$. It is known that M is β-homogeneous:

$$M(x) = |x|^\beta M(x/|x|), \qquad x \in \mathbb{R}^d \setminus \{0\},$$

where $\beta = \beta(d, \alpha, \Theta) \in (0, \alpha)$. For instance, $\beta = \alpha/2$ for the half-space, i.e. for $\Theta = \pi/2$. The homogeneity degree β is crucial for precise asymptotics of nonnegative harmonic functions of $\Delta^{\alpha/2}$ in cones. Also, the critical exponent of integrability of the first exit time of the corresponding isotropic α-stable Lévy processes from Γ_θ is simply $p_0 = \beta/\alpha$, which is a long-standing motivation to study β. In fact, the Martin, Green and heat kernels of $\Delta^{\alpha/2}$ for Γ_Θ enjoy explicit elementary estimates in terms of β. Denote $B_{d,\alpha} = \Gamma\left(\frac{d+\alpha}{2}\right) \pi^{-3/2} \sin\left(\frac{\pi\alpha}{2}\right) B\left(1 + \frac{\alpha}{2}, \frac{d-1}{2}\right) / \Gamma\left(\frac{d-1+\alpha}{2}\right)$, where Γ and B are the Euler gamma and beta functions, respectively. Here is our main result:

$$\beta(d, \alpha, \Theta) \asymp \alpha - B_{d,\alpha} \Theta^{d-1+\alpha} \quad \text{as} \ \ \Theta \to 0.$$

This resolves a decade-old puzzle. An application to the classical Laplacian in a complement of a plane slit by a cone is also given. The paper is available on arXiv. It is a joint work with Bartłomiej Siudeja (University of Oregon) and Andrzej Stós (Université Blaise Pascal, Clermont-Ferrand).

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Gaussian estimates for Schrödinger perturbations

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A perturbation series is an explicit method of constructing new semigroups or fundamental solutions. It is thus of the interest to obtain its upper and lower bounds.

We propose a new general method of estimating Schröodinger perturbations of transition densities using an auxiliary transition density as a majorant of the perturbation series. We present applications to Gaussian bounds by proving an optimal 4G Theorem for the Gaussian kernel, the inequality which is a non-trivial extension of the so called 3G or 3P Theorem (as well known, 3P fails in its primary form for the Gaussian kernel). Further applications concern transition denisty of 1/2 stable subordinator.

The talk is based on the paper [1] and other recent results.

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Discrete-time insurance models and their stability

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It is well known that the classical Cramér-Lundberg model and its numerous modifications attract attention of many researchers since the beginning of the last century. The primary task of any insurance company is satisfaction of the customers claims therefore the main subject of investigation was and still is the ruin probability, see, e.g., Yang et al. [5]. That means the reliability approach dominates in actuarial mathematics.

However, being a corporation, the insurance company has the secondary but very important task of paying dividends to its shareholders. The seminal paper by De Finetti [3] introduced the dividend problem, thus initiating the cost approach, see also, Bulinskaya [1]. To avoid ruin insurer can use reinsurance and capital injections. In such a situation insurers (or the company shareholders) are interested in minimization of additional costs. This research direction became very popular in the last decade. We are going to study some new models of this type. Since reinsurance treaties are usually bought at the end of financial year it is reasonable to consider discrete time models, see, e.g., the review by Li et al. [4] and references therein.

One of the models treated in the talk takes into account various types of reinsurance (proportional and nonproportional ones) and capital injections entailed by bank loans and/or investments in risky assets. It is also supposed that the claim process is described by a sequence of random variables. In the simplest case we deal with nonnegative independent identically distributed random variables. Insurance and reinsurance premiums are calculated according to the mean value principle with safety loading λ and μ respectively.

At first we establish the optimal control, that is, the parameters of reinsurance treaty minimizing the objective function (total expected discounted costs during the planning horizon of n periods).

It is necessary as well to verify the model stability with respect to small fluctuations of system parameters and perturbations of underlying processes. So, the next step is to carry out the sensitivity analysis, see, e.g., Bulinskaya [2]. We use the local and global technique, in particular, provide the global sensitivity indices GI of parameters λ and μ. Two examples of the graphs for these indices as functions of parameters relative errors k, calculated by means of Wolfram Mathematica 8 software, are given by Fig. 1 and Fig. 2.

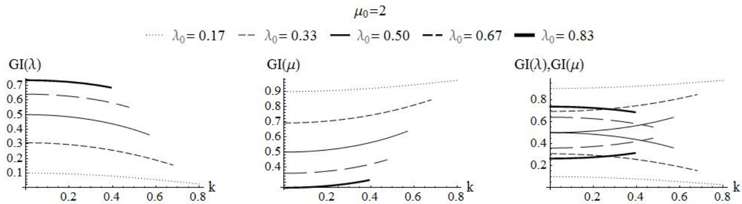

Figure 1: Sensitivity indices, μ fixed.

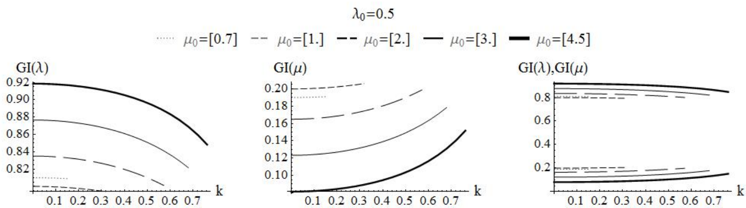

Figure 2: Sensitivity indices, λ fixed.

To estimate the impact of claim process distribution on optimal control we use probability metrics introduced in Zolotarev [6] and various stochastic orders.

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Identification of significant factors

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In a number of stochastic models a random response variable Y depends on some (in general random) factors $X_1, \ldots, X_n$. In medical and biological studies Y can describe the health state of a patient and $X = (X_1, \ldots, X_n)$ includes the genetic factors characterizing changes in DNA structure, e.g., SNP (single nucleotide polymorphisms), and non-genetic ones, for example, arterial pressure, obesity index etc. The challenging problem is to determine the collection of indices $\alpha = (k_1, \ldots, k_r)$ where $1 \le k_1 < \ldots < k_r \le n$ such that Y depends "essentially" on $X_\alpha = (X_{k_1}, \ldots, X_{k_r})$ and the impact of complementary set of factors X_i, $i \notin \{k_1, \ldots, k_r\}$, can be viewed as negligible in a sense. This problem is important for analysis of risk factors of complex diseases, for instance, diabetes, myocardial infarction and others. Often one employs the binary response variable Y taking values -1 and 1. In medicine $Y = 1$ and $Y = -1$ can correspond to the states *sick* or *healthy*, respectively. In pharmacology two values of Y can indicate efficiency or non-efficiency of some drug. Clearly, for many applications it is important to consider nonbinary response. The identification of significant factors when Y takes values in a finite set is the goal of this talk based on papers [1]-[3].

There are complementary approaches to the problem mentioned above. Among diverse statistical methods employed here we mention the principle component analysis, logic and logistic regressions, LASSO and various machine learning techniques. We concentrate on the new MDR (multifactor dimensionality reduction) method developed in [1]–[3]. The quality of Y prediction by means of $f(X_\alpha)$, where f is nonrandom function, is described by the specified error functional $Err(f)$. It involves a penalty function ψ allowing to consider the importance of predicting different values of Y. The joint law of the response and factors is unknown. Therefore it is natural that statistical inference is based on the error functional estimates constructed by prediction algorithm (involving i.i.d. observations (Y^i, X^i) where $Law(X^i, Y^i) = Law(X, Y)$ for $i = 1, \ldots, N$) and $K-$cross-validation procedure.

One of our main results is the criterion of strong consistency of the proposed estimates enabling one to identify the collection of significant factors. The strong consistency plays an important role as the comparison of estimated prediction errors for functions of different collections of factors is performed. Moreover, statistical estimates of unknown penalty function are used as well. We also introduce the regularized versions of these estimates and establish for them the central limit theorem (CLT). A statistical variant of our CLT allows us to indicate the approximate confidence intervals for unknown error functional. To conclude we discuss the importance of collections of factors following Schwender et al. [4].

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On order flow modeling with Cox processes

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The availability of high frequency data on transactions, quotes and order flow in electronic order-driven markets has revolutionized data processing and statistical modeling techniques in finance and brought up new theoretical and computational challenges. Market dynamics at the transaction level cannot be characterized solely in terms the dynamics of a single price and one must also take into account the interaction between buy and sell orders of different types by modeling the order flow at the bid price, ask price and other levels of the *limit order book* - aggregated collection of outstanding orders from buyers and sellers. Dynamics of limit order book is defined by three types of orders: *limit* orders (intention to buy or sell at a certain price), *market* orders (intention to buy or sell at the best price immediately) and *cancel* orders (which cancel one of previously placed limit order).

In [1] we use compound Cox processes to model order flows taking into account the stochastic nature of its intensities. We fix a time interval $[0;T]$ which is short enough so that the average parameters of the distributions of sizes of incoming orders could be assumed known within this interval. We consider the *order imbalance process* ([2]) in the form

$$Q(t) = \sum_{i=1}^{N^+(t)} X_i^+ - \sum_{j=1}^{N^-(t)} X_i^-,$$

where X_i^+ are identically distributed sizes of buy orders, X_i^- are identically distributed sizes of sell orders, $N^+(t)$ and $N^-(t)$ are the counting processes for the arrivals of buy and sell orders. The stochastic structure of the intensities of these counting processes is modeled by doubly stochastic Poisson processes (Cox processes) $N^+(t) = N_1^+(\Lambda^+(t))$ and $N^-(t) = N_1^-(\Lambda^-(t))$, where $N_1^+(t)$ and $N_1^-(t)$ are two standard Poisson processes with unit intensities, $\Lambda^+(t)$ and $\Lambda^-(t)$ are some non-decreasing right-continuous functions such that $\Lambda^+(0) = \Lambda^-(0) = 0$ and $\Lambda^+(\infty) = \Lambda^-(\infty) = \infty$.

In real practice the intensities of order flows are not independent, so we assume that $\Lambda^+(t) = \alpha^+(t)L(t)$ and $\Lambda^-(t) = \alpha^-(t)L(t)$, where $L(t)$ is a random measure playing the role of external informational background, $\alpha^+(t)$ and $\alpha^-(t)$ are multipliers describing the reaction degree of buyers and sellers to this background.

LEMMA 1. *If the random variables* $X_1^+, X_2^+, \ldots, X_1^-, X_2^-, \ldots$ *and the stochastic processes* $N_1^-(t)$, $N_1^+(t)$ *and* $L(t)$ *are independent, then for each* $t \geqslant 0$ *the order imbalance process* $Q(t)$ *has a compound mixed Poisson distribution:*

$$\mathsf{P}\big(Q(t) < x\big) = \mathsf{P}\big(\textstyle\sum_{j=1}^{N(t)} X_{t,j} < x\big), \quad x \in \mathbb{R},$$

where $N(t) = N_1\big(\Lambda(t)\big)$, $\Lambda(t) = \big(\alpha^+(t) + \alpha^-(t)\big)L(t)$, $N_1(t)$ *is a standard Poisson process independent of the process* $L(t)$ *and* $X_{t,1}, X_{t,2}, \ldots$ *are identically distributed random variables with the common characteristic function*

$$\mathfrak{f}_t(s) \equiv \mathsf{E}e^{isX_{t,1}} = \frac{\alpha^+(t)\mathfrak{f}^+(s)}{\alpha^+(t) + \alpha^-(t)} + \frac{\alpha^-(t)\mathfrak{f}^-(-s)}{\alpha^+(t) + \alpha^-(t)}, \quad s \in \mathbb{R},$$

where $\mathfrak{f}^+(s)$ *and* $\mathfrak{f}^-(s)$ *are the characteristic functions of* X_1^+ *and* X_1^- *respectively. Moreover, for each* $t \geqslant 0$ *the random variables* $N_1(t), \Lambda(t), X_{t,1}, X_{t,2}, \ldots$ *are independent.*

Consider a sequence of order flow imbalance processes of the form

$$Q_n(t) = \textstyle\sum_{j=1}^{N_1^{(n)}(\Lambda_n(t))} X_{t,j}^{(n)}, \quad t \geqslant 0,$$

For simplicity we will write Q_n, N_n, Λ_n and $X_{n,j}$ instead of $Q_n(t)$, $N_n(t)$, $\Lambda_n(t)$ and $X_{t,j}^{(n)}$ respectively.

THEOREM 1. *Assume that there exist an infinitely increasing sequence* $\{k_n\}_{n \geqslant 1}$ *of natural numbers and finite numbers* $\mu \in \mathbb{R}$ *and* $\sigma > 0$ *such that the randomized order sizes* $X_{n,j}$ *satisfy the condition*

$$\mathsf{P}(X_{n,1} + \ldots + X_{n,k_n} < x) \Longrightarrow \Phi\Big(\frac{x - \mu}{\sigma}\Big) \text{ when } n \to \infty.$$

The convergence

$$\mathsf{P}\big(Q_n < x\big) \Longrightarrow F(x)$$

takes place with some distribution function $F(x)$ *if and only if there exists a distribution function* $A(x)$ *such that* $A(0) = 0$, *the distribution function* $F(x)$ *is representable in the form*

$$F(x) = \int_0^\infty \Phi\Big(\frac{x - \mu z}{\sigma\sqrt{z}}\Big) dA(z),$$

and

$$\mathsf{P}(\Lambda_n < xk_n) \Longrightarrow A(x).$$

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Probability density function of myogram noise and its role in localization of brain activity sources

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Main task of our research was to explore characteristics of myogram rest domains and determine relevant parameters and distribution. Accelerometer signals were used to identify bounds of such rest intervals. These signal records, accelerometer and myogram, are usually recorded simultaneously so they can be easily fitted.

According to the obtained histograms an assumption was made that window variance of myogram responses within rest interval have gamma distribution with time-varying parameters. Probability density function in this case is:

$$f_X(x) = \begin{cases} (x-c)^{k-1} \frac{e^{-(x-c)/\theta}}{\theta^k\,\Gamma(k)}, & x \geqslant c \\ 0, & x < c \end{cases},$$

where $\Gamma(k)$ is the gamma function evaluated at k.

The mean and variance is defined as $k\theta + c$ and $k\theta^2$ respectively. Based on our distribution assumption the parameters of each rest domain were chosen. Figure 1 and Figure 2 show histograms of parameters k and θ.

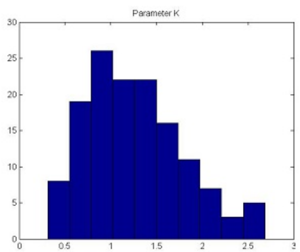

Figure 1: Parameter k.

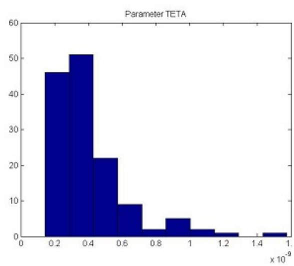

Figure 2: Parameter θ.

Myogram signals are associated with relevant magnetoencephalogram signals so an associative filter can be built up. Recognized characteristics of myogram noise makes it possible to refine the algorithm of reference points identification. These reference points are applied to magnetoencephalogram signals, that enables to select a principal sensor and utilize developed earlier algorithm based on Independent Component Analysis (ICA) and obtain analytical solutions of inverse problem (IP). In MEG context IP can be defined as:

$$B_t = GJ_t + N_t,$$

where: $B_t \in R_{Nsensors}$ is the random vector representing the measured data at time t; G is the lead-field matrix; $JtR_{Npoints}$ is the random vector representing the sources distribution at time t; $N_t \in R_{Nsensors}$ is the noise in the model.

The two main steps of proposed IP solving algorithm are:

- application ICA to raw MEG data, as a result decomposing relevant independent signal sources and separation multi-dipole model into several monodipole models;
- employing the analytical formula based on Biot Savart equation to obtained independent components as for monodipole models.

Hence assuming gamma distribution of myogram responses window variance within rest interval more precise model can be constructed to be dealt with. Our future investigation refers to improvement of rest bounds search algorithm, and also to refinement and fitting of distribution parameters.

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On asymptotic normality of risk estimate for Wavelet and Wavelet-Vaguelette decompositions of a signal with a correlated noise

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Statistical wavelet methods are widely used in the processing of noised signals and images, which are usually given as discrete observations:

$$Y_i = f_i + e_i. \tag{1}$$

Wavelet decomposition of a signal function $f \in L^2(\mathbb{R})$ is the series $f = \sum_{j,k\in Z}\langle f, \psi_{jk}\rangle\psi_{jk}$, where $\psi_{jk}(t) = 2^{j/2}\psi(2^j t - k)$, and $\psi(t)$ is a mother wavelet (the family $\{\psi_{jk}\}_{jk\in Z}$ forms an orthonormal basis in $L^2(\mathbb{R})$). The index j is called the scale, and the index k – the shift. We can choose such function ψ that has a sufficient number of vanishing moments and continuous derivatives, and also satisfies some other regularity conditions (see [1]). The signal function $f \in L^2(\mathbb{R})$ must also possess certain properties: it should have support in some finite interval and should be uniformly Lipschitz with an exponent $\gamma > 0$. We consider the model (1) with a correlated noise: $\{e_i, i \in \mathbb{Z}\}$ is a stationary Gaussian process with a covariance sequence $r_k = \mathbf{cov}(e_i, e_{i+k})$, a zero mean and a variance σ^2.

After discrete wavelet transform applied to (1) we may obtain two models depending on the rate of decay of the covariance sequence: short range and long range dependencies. The first one is (up to some constant) equivalent to the models with uncorrelated noise, studied in [2]. The second model is:

$$X_{jk} = \mu_{jk} + 2^{\frac{(J-j)(1-\alpha)}{2}} z_{jk}, \text{ where } z_{jk} = 2^{\frac{j(1-\alpha)}{2}} \int \psi_{jk} d\mathbf{B}_H, \tag{2}$$

$j = 1, \dots, J,\ k = 1, \dots, 2^j$, $0 < \alpha < 1$ is a decay parameter of the model, and μ_{jk} are discrete wavelet coefficients of the target function f (without noise).

Using a soft-thresholding procedure (see [2]), one can construct estimates for target functions(signals, images and so on). The presence of noise leads to the errors in these estimates. We can not calculate these errors strictly because they depend on unknown "clean" wavelet-coefficients, but we can estimate them:

$$\widehat{R}_J(f) = \sum_{j=0}^{J-1}\sum_{k=0}^{2^j-1} F[X_{jk}^2, T_j, \sigma_j],$$

where $F[x, T, \sigma] = (x - \sigma^2)\mathbf{1}(|x| \leqslant T^2) + (\sigma^2 + T^2)\mathbf{1}(|x| > T^2)$. Within the model (2) framework we proved that for $\alpha > 1/2$, $\gamma > (4\alpha - 2)^{-1}$ and the soft-thresholding procedure with a "universal" threshold $T_j = \sigma_j \sqrt{2 \ln 2^J}$ (σ_j is a variance of empirical wavelet-coefficients on the j-th scale) there is a convergence in distribution:

$$\frac{\widehat{R}_J(f) - R_J(f)}{D_J} \Rightarrow \mathbf{N}(0,1), \qquad J \to \infty, \text{ where } D_J^2 = C_\alpha 2^J, \tag{3}$$

and the constant C_α depends only on α and the chosen wavelet basis.

In addition, we proved the consistency of the risk estimate for the soft-thresholding with a "universal" threshold. For $0 < \alpha < 1$, $\gamma > 0$, and $b > 1 - \alpha + \alpha(2\gamma+1)^{-1}$ we have

$$\frac{\widehat{R}_J(f) - R_J(f)}{2^{bJ}} \xrightarrow{\text{P}} 0, \quad J \to \infty. \tag{4}$$

There is also a number of important applied problems where data is observed indirectly, for example, telecommunication traffic analysis, plasma physics, computer tomography and so on. They are described by the following data model:

$$Y_i = (Kf)_i + e_i, \tag{5}$$

where K is some linear homogeneous operator in $\mathbb{L}^2$ with a parameter β, f is a signal function, e_i is a correlated Gaussian noise with a zero mean. We employ "wavelet-like" functions $\{\xi_{jk}\}$ (vaguelettes, see [3]), such that $[Kf, \xi_{jk}] =< f, \psi_{jk} >$. Applying the discrete vaguelette transform, we obtain a model of discrete empirical vaguelette-coefficients:

$$X_{jk} = \mu_{jk} + 2^{J(1-\alpha)/2} w_{jk}, \text{ where } w_{j,k} = \int \xi_{jk} d\mathbf{B}_H, \tag{6}$$

$j = 1, \dots, J$, $k = 1, \dots, 2^j$, $0 < \alpha < 1$ is a model parameter, and μ_{jk} are discrete vaguelette-coefficients without a noise.

In the framework of this model we proved an asymptotic normality of the risk estimate for $\alpha + 2\beta > 1/2$, $\gamma > (4(\alpha+2\beta)-2)^{-1}$ and the soft-thresholding procedure with a "universal" threshold $T_j = \sqrt{2 \ln 2^j} \sigma_j$:

$$\frac{\widehat{R}_J(f) - R_J(f)}{D_J} \Rightarrow \mathbf{N}(0,1), \qquad J \to \infty, \qquad D_J^2 = \tilde{C} 2^{J(1+4\beta)}, \tag{7}$$

where $\tilde{C}$ depends only on α, β, and the chosen wavelet basis.

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The asymptotic behaviour of a random graph model

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A general random graph evolution mechanism is defined (see Fazekas and Porvázsnyik [4], Fazekas and Porvázsnyik [5]). The evolution is based on interactions of N vertices. Besides the interactions of the new vertex with the old ones, interactions among old vertices are also allowed. Moreover, both preferential attachment and uniform choice are possible. Our model is a generalization of the three-interactions model introduced in Backhausz and Móri [3].

A vertex in our graph is characterized by its degree and its weight. The weight of a given vertex is the number of the interactions of the vertex. The asymptotic behaviour of the graph is studied. Scale-free properties both for the degrees and the weights are proved. A random graph is called scale-free, if $p_k \sim Ck^{-\gamma}$, as $k \to \infty$, where $p_1, p_2, \dots$ is the asymptotic degree distribution of the graph. It turns out that in our model any exponent γ in $(2, \infty)$ can be achieved. Asymptotic results are obtained for the degree and the weight of a fixed vertex. Moreover, the maximal degree and the maximal weight are also studied. The proofs are based on discrete time martingale theory.

Some numerical results are also presented. Using computer simulation, our model is compared with the original Barabási-Albert preferential attachment rule and the Cooper-Frieze model, see Barabási and Albert [1], Cooper and Frieze [2].

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A practical solution of the fat tail problem in financial markets

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Logarithms of stock returns $Y_n = ln(S_n/S_0)$ over a span of n time intervals can be represented as a sum $Y_n = X_1 + \ldots + X_n$ of smaller-intervals$'$ log-returns $X_j, j = 1, \ldots, n$. In quantitative finance Y_n are usually assumed to be normally distributed. However, the observed tails of the empirical cumulative distribution function (c.d.f.) $F_n(t)$ of Y_n are usually much "fatter" than the tails of a normal distribution $\Phi(t)$ with the same parameters of location and scale. The "Fat Tail" phenomenon continued to puzzle for decades and attracted a lot of attention (the recent financial crisis is an example). In order to better explain the tails of F_n, researchers used a gamut of approaches: mixtures of normal distributions, stable distributions, different stochastic processes$'$ models ([9,10]). A particularly important case for investors is the annual log-return Y_n that consists of $n = 253$ daily log-returns X_j. Historical data observations show that (for properly normalized Y_{253}) the probabilities of six-standard-deviations (6σ) losses are quite substantial:

$$Pr\{Y_{253} < -6\sigma\} = F_{253}(-6\sigma) \approx 0.27 * 10^{-2}$$

while the corresponding value for a normal random variable is much smaller:

$$\Phi(-6\sigma) \approx 10^{-9}$$

The "fatness" ratio $F_{253}(-6\sigma)/\Phi(-6\sigma) \approx 2.7 * 10^6$ is huge, investors face 6-sigma losses *2.7 million* times more frequently than "promised" by CLT and the corresponding normal distribution. The following two major questions worry investors and specialists in the quantitative finance:

Question 1. Why the observed tails are so "fat"?

Question 2. Is there a way to make the tails thinner?

A short answer to the first question is that the observed tails are rather fit than fat. Indeed, rewrite $F_n(t)$ as:

$$F_n(t) = [F_n(t) - \Phi(t)] + \Phi(t)$$

A careful inspection that uses known estimates of the rate of convergence in CLT , shows that for $n = 253$ and for typical daily log-returns X_j in financial markets, the first term may dominate the second term by a factor of 10^7:

$$|F_n(t) - \Phi(t)| \gg \Phi(t)$$

It means that in order to properly estimate tails of $F_n(t)$, the $[F_n(t) - \Phi(t)]$ term may not be ignored, the rate of convergence in CLT should be taken into account very seriously. The use of known methods [1,2,3,6,8] that provide upper bounds for $|Fn(t) - \Phi(t)|$, combined with the assumption of independency of daily log-returns[5,7] allowed to build estimates[11] that proved to be consistent with the empirical market data.

The answer to the second question is yes, there is a way; the corresponding methodology[11] consists of the two parts:

Part 1. Truncate X_j, so that $|X_j| < M$. Practically, in finance, the truncation is equivalent to the construction of synthetic instruments through a self-financing strategy that includes options.

Part 2. Estimate the tails of the sums of the newly created bounded random variables using a proper set of concentration inequalities, including, in particular, Hoeffding′s inequality [4].

The observed tails of the sums of thus constructed instruments proved to be consistent with the estimates based on the proposed set of concentration inequalities and turned out to be much thinner than the tails of the sums of the original instruments.

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On Harnack inequality for unimodal Lévy processes

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We present some recent results about isotropic unimodal Lévy processes on $\mathbb{R}^d$ (i.e. rotation invariant Lévy process with the absolutely continuous Lévy measure which density is radially non-increasing). For instance the scale invariant Harnack inequality holds for harmonic functions with respect to an isotropic unimodal Lévy process with the characteristic exponent ψ satisfying some scaling condition. We derive sharp estimates of the potential measure and capacity of balls, and further, under the assumption that ψ satisfies the lower scaling condition, sharp estimates of the potential kernel of the underlying process. This allows us to establish the Krylov-Safonov type estimate. Further we show Hölder regularity properties for harmonic functions.

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Application of Dirichlet mixture of normals in growth curve models

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In our research, we present growth curve models with an auxiliary variable which contains an uncertain data distribution based on mixtures of standard components, such as normal distributions. The multimodality of the auxiliary random variable motivates and necessitates the use of mixtures of normal distributions in our model. We have observed that Dirichlet process priors, composed of discrete and continuous components, are appropriate in addressing the two problems of determining the number of components and estimating the parameters simultaneously and are especially useful in the aforementioned multimodal scenario. A model for the application of Dirichlet mixture of normals (DMN) in growth curve models under Bayesian formulation is presented and algorithms for computing the number of components, as well as estimating the parameters are also rendered. The simulation results show that our model gives improved goodness of fit statistics over models without DMN and the estimates for the number of components and for parameters are reasonably accurate.

We present growth curve models with auxiliary variables containing uncertain data distributions based on mixtures of standard components and using normal distributions in our simulation example. The results (from the algorithm we have developed) show that our model is useful in estimating the number of components in the mixture normals, the probabilities from which the auxiliary variables arise as well as the means of the normal distributions in the components of the mixture normals. The estimates and goodness of fit statistics (adjusted R2) in our simulating example show that models with DPP can outperform those models without DPP. We do not state the universal applicability of our model with only one simulation example but it suffices in showing the advantages of using our model especially in scenarios with some specific multimodal distributions from data.

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Fractional Laplacian with frift

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For $\alpha \in (1, 2)$ we consider the equation $\partial_t u = \Delta^{\alpha/2} u - b \cdot \nabla u$. We consider various classes of vector fields b and resulting fundamental solutions of the given equation $\tilde{p}$. As the result we show that $\tilde{p}$ is comparable to the transition density of the isotropic stable process.

Decay of eigenfunctions for nonlocal Schrödinger operators

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The decay of eigenfunctions at infinity for Schrödinger operators

$$H = -\Delta + V,$$

where V is a suitably chosen external potential, has been widely studied for many years. Its rate describes the localization of a quantum particle in a physical space. An explicit form of eigenfunctions is known only in few specific cases. Assuming that $\varphi \in L^2(\mathbf{R}^d)$ is an eigenfunction of H, i.e., $H\varphi = \lambda\varphi$, and

V is sufficiently regular potential, a basic question is how rapid is the decay of $\varphi(x)$ in function of V when $|x| \to \infty$. For pinning potentials, i.e., $V(x) \to \infty$ as $|x| \to \infty$, the decay is known to be typically exponential or faster. For instance, if $V(x) \asymp |x|^\beta$, $\beta \geq 1$, and φ_0 corresponds to the eigenvalue $\lambda_0 := \inf \mathrm{spec} H$ (the so-called ground state eigenfunction), we have

$$\varphi_0(x) \asymp |x|^{-\frac{\beta}{4}+\frac{(d-1)}{2}} e^{-\frac{2}{2+\beta}|x|^{1+\frac{\beta}{2}}}, \quad |x| \geq 1.$$

Similar questions, motivated by the problems in a relativistic quantum mechanics, appears in the case of the so-called nonlocal Schrödinger operators

$$H = -L + V,$$

where L is a nonlocal operator being the generator of the jump Lévy process. The most interesting example seems to be the relativistic Hamiltonian $L = -\sqrt{-\Delta + m} + m$, $m > 0$, the generator of the relativistic Lévy motion.

I will present the recent results on the pointwise bounds at infinity of the eigenfunctions for a wide class of operators L and signed potentials $V(x) \to \infty$, $|x| \to \infty$, possibly singular. These estimates explicitly depend on the density of the Lévy measure of the process generated by L and the growth of V at infinity. For the ground state eigenfunction (which is known to be strictly positive) they are even two-sided and sharp. Our methods are mainly probabilistic (stochastic Feynman-Kac type representation of the semigroup e^{-tH}) and are based on a precise analysis of the jumps of the process and some specific self-improving estimates iterated infinitely many times. These tools allow us to derive the sharp bounds even in the case of Lévy measures that are exponentially localized at infinity.

I will also discuss some interesting consequences and applications of these results such as properties of domination (the semigroup e^{-tH} and other eigenfunctions) by the ground state eigenfunction, the asymptotic behaviour of the semigroup e^{-tH} for large t (intrinsic ultracontractivity-type properties), the asymptotic behaviour of paths of the related ground state-transformed jump processes (integral tests of the Kolmogorov type, LILs etc.).

The talk is based on a joint work with J. Lőrinczi (Loughborough University).

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The bounds of the convergence rate for unreliable queuing network

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In this paper we consider a Jackson type network with unreliable devices. The network consists of $m, (m < \infty)$ nodes, each node is a queuing system of M/G/1 type. It is assumed that the flow of requests, coming into the network, is the Poisson process with parameter $\lambda(t)$. With probability r_{0i} the request is sent to the i- th node , $\sum_{i=1}^{m} r_{0i} \leq 1$, where it is proccessed with intensity $\mu_i(n_i)$, n_i - the number of requests in the i- th node . Devices in the network may break down or repair with some intensity, depending on the number of already broken down devices. Devices may break down and repair as an isolated event or in groups simultaneously. In this paper we will formulate results on the bounds of convergence rate for such network.

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Stable measure of dependence for network analysis

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Network models are popular tools for financial market analysis Tumminello M., Aste T., Matteo T.D., Mantegna R.N. [1], Boginsky V.,Butenko S.,Pardalos P. [2]. The network model is a complete weighted graph in which nodes corresponds to a stocks and weights of edges between nodes are equal to value of measure of similarity (dependence) of stocks behavior. The most popular measure of dependence of the random variables used in network analysis is the classic Pearson correlation. It is well known that for a multivariate normal distribution covariance matrix is a sufficient statistics Anderson T.W. [3]. However the assumption of multivariate normal distribution of real data is not satisfied. In particular multivariate distributions of real data of stock returns have a more heavy tails, than multivariate normal distribution Shiryaev A.N. [4].

In Bautin G.A., Kalyagin V.A., Koldanov A.P., Koldanov P.A., Pardalos P.M. [5] sign correlation is used as an alternative measure of dependence. This measure is based on the probability of coincidence of the random variables signs. It is shown that such a measure is appropriate for the market network analysis, has a simple interpretation, can be generalized to any number of random variables and has a connection to the Pearson correlation in the case of normal distribution. In Bautin G.A., Kalyagin V.A., Koldanov A.P. [6] these measures are compared for different models of financial market.

In the present report connection between Pearson correlation and sign correlation is investigated for elliptically contoured distributions. A mixture of multivariate normal distribution and multivariate Student distribution is considered as a model of simultaneous behavior of stock returns of financial market. Stability of statistical estimations of Pearson and sign correlations is compared for the model. Some structural characteristics of complete weighted graph, namely minimal spanning tree Tumminello M. [1], market graph Boginsky V., Butenko S., Pardalos P. [2], are considered. Construction problem of these characteristics as multiple decision statistical procedure is formulated Koldanov A.P., Koldanov P.A., Kalyagin V.A., Pardalos P.M. [7]. Stability of such procedures is measured by conditional risk Lehmann E.L., Romano J.P. [8]. It is shown that statistical procedures based on sign correlation are stable with respect to parameters of mixture of multivariate normal distribution and multivariate Student distribution.

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Multivariate CAPM: Estimation and Testing

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Classical linear model is not good for analysis of modern securities markets. In classical model the error term has multivariate normal distribution with zero mean and covariance matrix which is proportional to unit matrix, i.e. the components are uncorrelated and has the equal variances. Many real measurements show that it is not true.

In our report we consider the following model:

$$Y_t = X_t \cdot \theta + \varepsilon_t \ , \ t = 1, 2, \ldots .$$

We assume that the error term ε_t has the properties:

1) ε_t has multivariate Student distribution with dependent components,

2) random vectors ε_t follow multivariate GARCH model,

3) time series ε_t has the property of long range dependence.

This model is very different from classical one and the ordinary statistical procedures don't work.

We propose some new methods for estimation of parameters of this model and testing hypothesis and investigate their properties.

Next we apply this model for analysis of Russian securities market.

Analogous models were considered in [1] and [2].

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Estimation of ruin probability in the collective risk model with investments

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In our model we assume that insurance company capital $R(t)$ at the moment t can be discribed by classical Cramer-Lundberg process:

$$R(t) = u + ct - \sum_{j=1}^{N(t)} Z_j = u + ct - P(t) \ , \tag{1}$$

where u – size of initial capital of the company, $(N(t), t \geq 0)$ – homogeneous Poisson process, $\{Z_j\}$ – sequence of independent identically distributed positive random variables.

As usually we assume that the insurance company invests its capital in risk and riskless securities. The dynamics of these securities is described by following equations:

$$dS(t) = S(t)(\mu dt + \sigma dW(t)) \ , \tag{2}$$

$$dB(t) = rB(t)dt \ , \tag{3}$$

where $S(t)$ – the price of risky securities at the t, μ – mean return, σ – volatility, $W(t)$ – standard Brownian motion, $B(t)$ – the price of bonds at the moment t, r – riskless rate $(0 < r < \mu)$.

We assume that insurance company invests the part $\alpha(t)$ in risky securities and the part $1 - \alpha(t)$ in bonds. Then we have the following equation for the capital of company:

$$dX(t) = [\alpha(t)\mu + (1 - \alpha(t))rdt + \alpha(t)\sigma dW(t)] \cdot X(t) + dR(t) \ , \ X(0) = u \ . \tag{4}$$

In what follows we consider the case $\alpha(t) = \alpha = const$ and denote $\beta = \alpha\mu + (1 - \alpha)r$, $\gamma = \alpha\sigma$. Then equation (4) can be written in the form:

$$dX(t) = [\beta dt + \gamma dW(t)] \cdot X(t) + dR(t) \ , \ X(0) = u \ . \tag{5}$$

Now consider the auxiliary equation:

$$dC(t, s) = [\beta dt + \gamma dW(t)] \cdot C(t, s) \ , \ t > s, \ C(s, s) = 1 \ . \tag{6}$$

The solution of equation has the form:

$$C(t, s) = \exp[(\beta - \gamma^2/2)(t - s) + \gamma(W(t) - W(s))] \ , \ t \geq s \ . \tag{7}$$

(see [1]).

We show that the solution of equation (5) can be written in the form:

$$X(t) = C(t,0)u + c \cdot \int_0^t C(t,s)ds - \sum_{j=0}^{N(t)} Z_j \cdot C(t,\nu_j) \ , \qquad (8)$$

where ν_j are the moments of jumps of the process $(N(t), t \geq 0)$.

In the classical Cramer-Lundberg model without investments for ruin probability $\psi(u)$ was obtained the following representation:

$$\psi(u) = \frac{e^{-R\cdot u}}{M\left(e^{-R\cdot R(\tau)}|\tau < \infty\right)} \ , \qquad (9)$$

where τ is the moment of ruin and $R > 0$ is the positive solution of the equation:

$$\lambda + c \cdot r = \lambda \cdot M_Z(r) \ , \qquad (10)$$

here $M_Z(r)$ – moment generating function of r.v. Z_j. (see [2]).

From this result we get well known Lundberg inequality:

$$\psi(u) \leq e^{-R\cdot u} \ .$$

In our report using paper [3] we propose some lower and upper estimates for $\psi(u)$.

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Estimation of ruin probability in multivariate collective risk model

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We consider the following multivariate analog of classical collective risk model (see [1]):

$$\vec{U}(t) = (U_1(t), \ldots, U_m(t)) = \vec{u} + \vec{c} \cdot t - \vec{S}(t), t \geqslant 0,$$

where $\vec{u} = (u_1, \ldots, u_m))$, $\vec{c} = (c_1, \ldots, c_m) \in R^m$, $c_k > 0$ for all $k = \overline{1, m}$ and $\vec{S}(t) = (S_1)t), \ldots, S_m(t)$ is the payments process.

We introduce the multivariate index $i = (i_1, \ldots, i_m)$, whose components i_k receive two values: the value 1 when there are claims of kth type and the value 0 otherwise. Let us denote by I the set of all possible values of the index i and by I_k its subset that includes only such i whose the kth component is equal to 1: $I_k = \{i \in I : i_k = 1\}$.

For each index i, there is a random process $N^{(i)}(t), t \geq 0$, representing the number of insurance cases up to the moment t whose claims have a structure corresponding to the index i. For different i $N^{(i)}(t)$ are assumed to be independent Poisson processes with parameters $\lambda^{(i)}$. Then the vector counting process is defined by the following rule:

$$\vec{N}(t) = (N_1(t), \ldots, N_m(t)) = (\sum_{i \in I_1} N^{(i)}(t), \ldots, \sum_{i \in I_m} N^{(i)}(t)) \ .$$

Let $(X_j^{(i)}), j \geqslant 1$ be a sequence of independent and identically distributed random vectors in R_+^m, $(\varepsilon_j, j \geqslant 1)$ be a sequence of independent random variables which take their values in I and $P(\varepsilon_j = i) = \frac{\lambda^{(i)}}{\lambda}$.

If $N(t) = \sum\limits_{i \in I} N^{(i)}(t)$ then it is easy to see that

$$\vec{N}(t) = \sum_{j=1}^{N(t)} \varepsilon_j \ .$$

Now we define the payments process in the following form:

$$S_k(t) = \sum_{j=1}^{N(t)} \sum_{i \in I_k} I(\varepsilon_j = i) \cdot X_{j,k}^{(i)} \ .$$

Denote

$$X_{j,k}^* = \sum_{i \in I_k} I(\varepsilon_j = i) \cdot X_{j,k}^{(i)} \ .$$

Let $t_j = s \cdot r_j$, i.e. $\vec{t} = (t_1, t_2, \ldots, t_m)^T = (s \cdot r_1, s \cdot r_2, \ldots, s \cdot r_m)^T$, where $r_1 < r_2 < \ldots < r_m$.

First we prove that random vector

$$\vec{S}^*(\vec{t}) = \frac{\vec{S}(\vec{t}) - M(X_j^*) \circ r \cdot \lambda \cdot s}{\sqrt{\lambda \cdot s}} \ ,$$

where $a \circ b = (a_1 \cdot b_1, \ldots, a_m \cdot b_m)^T$, has asymptotically $(s \to \infty)$ multivariate normal distribution with zero mean and covariance matrix Σ_0, whose elements have the form $\sigma_{p,q} \cdot \min(r_p, r_q)$.

In one dimensional model it has been proved that ruin probability $\psi(u,t)$ can be approximated by

$$\Phi\left(\frac{t-u\cdot y_0}{\sqrt{u\cdot v_0}}\right)\cdot C\cdot e^{-R\cdot u}\ ,$$

as $u,t\to\infty$, under condition that the quantity $(t-u\cdot y_0)/\sqrt{u\cdot v_0}$ is bounded, where y_0, v_0, C, R are explicitely calculated constants (see [2], p. 137-141).

We get the analogous result in our multivariate model.

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On the limit distributions of the maximum tree size in a conditional Poisson Galton–Watson forest

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We consider the set of realizations of a subcritical or critical homogeneous Galton–Watson process starting with N particles such that the number of offspring of each particle has a Poisson distribution with parameter λ. This set is infinite and consists of rooted trees with a finite number of vertices. The probability distribution on this set is induced by the branching process. Such random forests are known as Galton–Watson forests. Let $\eta_{(N)}$ denote the maximum tree size in a Galton–Watson forest. As $N\to\infty$ for a subset of trajectories with a known identical number of vertices limit distributions of $\eta_{(N)}$ were obtained by Pavlov [1] using a generalized allocation scheme (Kolchin, [2]). We derived similar results for a subset of trajectories such that the number of vertices does not exceed n with different behavior of parameters λ and n. In particular, the following assertion is true.

Theorem. *Let* $N,\ r\to\infty$ *such as* $N\lambda^{r-1}e^{(1-\lambda)r}/\sqrt{2\pi r^3}\to\alpha$, *where* α *is a positive number,* $N-n(1-\lambda)\leqslant C\sqrt{N}$, $0\leqslant C<\infty$, $0<\lambda_1\leqslant\lambda\leqslant\lambda_2<1$. *Then for any fixed* k

$$\mathbf{P}\left\{\eta_{(N)}\leqslant r+k\right\}\to\exp\left\{-\frac{\alpha\left(\lambda e^{1-\lambda}\right)^{k+1}}{1-\lambda e^{1-\lambda}}\right\}.$$

For the conditional random forests in question the proved theorems generalize the results obtained by Chuprunov and Fazekas [3].

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Asymptotics for the estimation of the offspring means in critical two-type GWI processes

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The model is as follows. For each $k, j \in \mathbb{Z}_+$ and $i, \ell \in \{1, 2\}$, the number of individuals of type i in the k^{th} generation will be denoted by $X_{k,i}$, the number of type ℓ offsprings produced by the j^{th} individual who is of type i belonging to the $(k-1)^{\text{th}}$ generation will be denoted by $\xi_{k,j,i,\ell}$, and the number of type i immigrants in the k^{th} generation will be denoted by $\varepsilon_{k,i}$. Then

$$\begin{bmatrix} X_{k,1} \\ X_{k,2} \end{bmatrix} = \sum_{j=1}^{X_{k-1,1}} \begin{bmatrix} \xi_{k,j,1,1} \\ \xi_{k,j,1,2} \end{bmatrix} + \sum_{j=1}^{X_{k-1,2}} \begin{bmatrix} \xi_{k,j,2,1} \\ \xi_{k,j,2,2} \end{bmatrix} + \begin{bmatrix} \varepsilon_{k,1} \\ \varepsilon_{k,2} \end{bmatrix}, \qquad k \in \mathbb{N}.$$

We distinguish 3 cases based on the spectral radius of the offspring mean matrix

$$m_\xi := \begin{bmatrix} \mathbb{E}(\xi_{1,1,1,1}) & \mathbb{E}(\xi_{1,1,1,2}) \\ \mathbb{E}(\xi_{1,1,2,1}) & \mathbb{E}(\xi_{1,1,2,2}) \end{bmatrix}.$$

We focus our attention to the crtical case, that is when the spectral radius of the above matrix equals 1. We propose an estimate for m_ξ based on the conditional least squares method. We examine the asymptotic properties of the estimates. We also discuss the possibility of applying the same method for a more general model.

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Some product representations for random variables with Weibull distribution and their applications

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Let $\gamma > 0$. The distribution of the random variable W_γ:

$$\mathsf{P}\big(W_\gamma < x\big) = \begin{cases} 1 - e^{-x^\gamma}, & x \geqslant 0, \\ 0, & x < 0, \end{cases} \tag{1}$$

is called the Weibull distribution with the shape parameter γ. It is called after Waloddi Weibull (1887 – 1979), the Swedish scientist, who used this distribution in 1939 for the statistical analysis of the strength of materials [1, 2] and studied the properties of this distribution in [3]. However, Weibull was not the first to study distribution (1). This distribution was introduced in 1927 by Maurice Fréchet [4] as a limit law for extreme order statistics and used by Paul Rosin, Erich Rammler and Karl Sperling in 1933 [5, 6] and John Godolphin Bennett in 1936 [7] as a model for the coal particle size distribution.

The Weibull distribution is widely used in various applied problems, see, e. g., [8–12]. Main applications of this distribution deal with survival analysis and reliability theory where it is used as a lifetime distribution. It is also worth noticing that in the papers [13, 14] the Weibull distribution was used as a successful model for the asset returns regularities.

It is obvious that W_1 is a random variable with the standard exponential distribution function

$$\mathsf{P}(W_1 < x) = E(x) \equiv \big[1 - e^{-x}\big]\mathbf{1}(x \geqslant 0).$$

The Weibull distribution with $\gamma = 2$

$$\mathsf{P}(W_2 < x) = \big[1 - e^{-x^2}\big]\mathbf{1}(x \geqslant 0)$$

is called the Rayleigh distribution after John William Strutt, lord Rayleigh who introduced this distribution within the framework of the problem of addition of a large number of vibrations of the same pitch and of arbitrary phase [15].

Let X be a random variable with the standard normal distribution function $\Phi(x)$:

$$\mathsf{P}(X < x) = \Phi(x) = \int_{-\infty}^{x} \varphi(z)dz, \quad \varphi(x) = \frac{1}{\sqrt{2\pi}}e^{-x^2/2}, \quad x \in \mathbb{R}.$$

Let $\Psi(x)$, $x \in \mathbb{R}$, be the distribution function of the maximum of the standard Wiener process on the unit interval,

$$\Psi(x) = 2\Phi\big(\max\{0, x\}\big) - 1, \quad x \in \mathbb{R}.$$

It is easy to see that $\Psi(x) = \mathsf{P}(|X| < x)$. Therefore sometimes $\Psi(x)$ is called the half-normal distribution function.

The symbol $\stackrel{d}{=}$ will denote the coincidence of distributions.

LEMMA 1. *The relation*

$$W_1 \stackrel{d}{=} \sqrt{2W_1}|X|$$

holds, where the random variables on the right-hand side are independent.

THEOREM 1. *Let $\gamma > 0$. For any $k \in \mathbb{N}$ we have*

$$W_\gamma \stackrel{d}{=} 2^{(2^{k-1}-1)(2^k\gamma)^{-1}} W_1^{2^{-k}\gamma^{-1}} \Big(\prod\nolimits_{m=1}^{k} |X_m|^{1/2^{m-1}}\Big)^{1/\gamma},$$

where the random variables on the right-hand side are independent and $X_1, X_2, \ldots$ have the same standard normal distribution.

COROLLARY 1. *Let $\gamma > 0$. We have*

$$W_\gamma \stackrel{d}{=} \Big(2\prod\nolimits_{m=1}^{\infty} |X_m|^{1/2^{m-1}}\Big)^{1/\gamma},$$

where the random variables $X_1, X_2, \ldots$ are independent and have the same standard normal distribution..

COROLLARY 2. *For any $k \in \mathbb{N}$ we have*

$$W_1 \stackrel{d}{=} 2^{(2^{k-1}-1)2^{-k}} W_1^{1/2^k} \prod\nolimits_{m=1}^{k} |X_m|^{2^{1-m}}, \qquad W_1 \stackrel{d}{=} 2\prod\nolimits_{m=1}^{\infty} |X_m|^{2^{1-m}},$$

$$W_2 \stackrel{d}{=} 2^{(2^{k-1}-1)2^{-k-1}} W_1^{1/2^{k+1}} \prod\nolimits_{m=1}^{k} |X_m|^{2^{-m}}, \qquad W_2 \stackrel{d}{=} \sqrt{2}\prod\nolimits_{m=1}^{\infty} |X_m|^{2^{-m}},$$

where the random variables on the right-hand sides are independent and $X_1, X_2, \ldots$ have the same standard normal distribution.

By $G_{\alpha,\theta}(x)$ and $g_{\alpha,\theta}(x)$ we will respectively denote the distribution function and the density of the strictly stable law with the characteristic exponent α and parameter θ corresponding to the characteristic function

$$\mathfrak{f}_{\alpha,\theta}(t) = \exp\Big\{-|t|^\alpha \exp\Big\{-\frac{i\pi\theta\alpha}{2}\mathrm{sign} t\Big\}\Big\}, \quad t \in \mathbb{R},$$

where $0 < \alpha \leqslant 2$, $|\theta| \leqslant \theta_\alpha = \min\{1, \frac{2}{\alpha} - 1\}$ (see, e. g., [16]).

In order to prove that any Weibull distribution with parameter $\gamma \in (0, 1]$ is a scale mixture of half-normal laws we first prove that any Weibull distribution with parameter $\gamma \in (0, 2]$ is a scale mixture of Rayleigh distributions.

LEMMA 2. *For any* $\gamma \in (0,2]$ *we have*

$$W_\gamma \stackrel{d}{=} W_2\sqrt{\eta_{\gamma/2}},$$

where $\eta_{\gamma/2} = 2\zeta_{\gamma/2,1}^{-1}$, *and* $\zeta_{\gamma/2,1}$ *is a random variable with one-sided strictly stable density* $g_{\gamma/2,1}(x)$ *independent of* W_2.

THEOREM 2. *For any* $\gamma \in (0,1]$, *the Weibull distribution with parameter* γ *is a scale mixture of half-normal laws*:

$$W_\gamma \stackrel{d}{=} |X|\sqrt{2W_1\eta_\gamma^2},$$

where $\eta_\gamma = 2\zeta_{\gamma,1}^{-1}$, *and* $\zeta_{\gamma,1}$ *is a random variable with one-sided strictly stable density* $g_{\gamma,1}(x)$, *moreover, the random variables on the right-hand side are independent.*

COROLLARY 3. *The Weibull distribution with parameter* $\alpha = \gamma/2 \in (0,1]$ *is a mixed exponential distribution*:

$$e^{-x^\alpha} = \mathsf{P}(W_\alpha > x) = \mathsf{P}(W_1 > \tfrac{1}{2}\zeta_{\alpha,1}x) = \int_0^\infty e^{-\frac{1}{2}zx} g_{\alpha,1}(z)dz, \quad x \geqslant 0.$$

REMARK 1. The case $\gamma \in (0,1]$ is of special interest since the Weibull distributions with such parameters occupy an intermediate position between the laws with exponentially decreasing tails and Zipf–Pareto-type heavy-tailed distributions.

Theorem 2 implies that if $\gamma \in (0,1]$, then

$$\mathsf{P}(W_\gamma < x) = \mathsf{E}\Psi\Big(\frac{x}{\sqrt{2W_1\eta_\gamma^2}}\Big) = \int_0^\infty \Psi\Big(\frac{x}{\sqrt{y}}\Big)dH_\gamma(y), \quad x \in \mathbb{R},$$

where

$$H_\gamma(y) = \mathsf{P}(2W_1\eta_\gamma^2 < y) = \mathsf{P}\big(W_1 < \tfrac{1}{8}y\zeta_{\gamma,1}^2\big) = \\ = 1 - \int_0^\infty \exp\big\{-\tfrac{1}{8}yz^2\big\}g_{\gamma,1}(z)dz, \quad y \geqslant 0. \tag{2}$$

COROLLARY 4. *Let* $\zeta_{\alpha,1}$ *be a random variable with the one-sided strictly stable distribution with characteristic exponent* $\alpha = 2^{-k}$, $k \in \mathbb{N}$. *Then*

$$\zeta_{\alpha,1} \stackrel{d}{=} \Big(2^{2^{k-2}-1}\prod\nolimits_{m=1}^k |X_m|^{2^m}\Big)^{-1}.$$

Let $\gamma > 0$. The symmetric two-sided Weibull distribution with parameter γ is the distribution of the random variable $\widetilde{W}_\gamma$:

$$\mathsf{P}\big(\widetilde{W}_\gamma < x\big) = \tfrac{1}{2}e^{-|x|^\gamma}\mathbf{1}(x<0) + \big[1 - \tfrac{1}{2}e^{-x^\gamma}\big]\mathbf{1}(x \geqslant 0). \tag{3}$$

Distribution (3) was introduced in [17] as a heavy-tailed model for financial risks. Further generalizations and references can be found in [18, 19].

It is easy to see that if W_γ is a random variable with Weibull distribution (1) and Z is a random variable independent of W_γ taking the values -1 and $+1$ with probabilities $\frac{1}{2}$ each, then $\widetilde{W}_\gamma \stackrel{d}{=} ZW_\gamma$ and hence, $|\widetilde{W}_\gamma| \stackrel{d}{=} W_\gamma$.

From theorem 2 it obviously follows that

$$W_\gamma \stackrel{d}{=} X\sqrt{2W_1\eta_\gamma^2},$$

where $\eta_\gamma = 2\zeta_{\gamma,1}^{-1}$, and $\zeta_{\gamma,1}$ is a random variable with the one-sided strictly stable density $g_{\gamma,1}(x)$, moreover, the random variables on the right-hand side are independent.

Corollary 5. *For any $\gamma \in (0,1]$, the symmetric two-sided Weibull distribution with parameter γ is a scale mixture of normal laws*:

$$\mathsf{P}(\widetilde{W}_\gamma < x) = \mathsf{E}\Phi\Big(\frac{x}{\sqrt{2W_1\eta_\gamma^2}}\Big) = \int_0^\infty \Phi\Big(\frac{x}{\sqrt{y}}\Big)dH_\gamma(y), \quad x \in \mathbb{R},$$

where

$$H_\gamma(y) = 1 - \int_0^\infty \exp\big\{-\tfrac{1}{8}yz^2\big\}g_{\gamma,1}(z)dz, \quad y \geqslant 0.$$

It is obvious that $\widetilde{W}_1$ is a random variable with the Laplace distribution

$$L(x) \equiv \mathsf{P}(\widetilde{W}_1 < x) = \tfrac{1}{2}e^x\mathbf{1}(x<0) + \big[1 - \tfrac{1}{2}e^{-x}\big]\mathbf{1}(x \geqslant 0).$$

It is easy to see that

$$\widetilde{W}_1 \stackrel{d}{=} X\sqrt{2W_1},$$

with the random variables on the right-hand side being independent (see, e. g., [20], p. 578-579). Then Corollary 5 implies

Corollary 6. *For any $\gamma \in (0,1]$, the symmetric two-sided Weibull distribution with parameter γ is a scale mixture of the Laplace distributions*:

$$\mathsf{P}(\widetilde{W}_\gamma < x) = \mathsf{E}L\big(\tfrac{1}{2}x\zeta_{\gamma,1}\big) = \int_0^\infty L(\tfrac{1}{2}xy)g_{\gamma,1}(y)dy, \quad x \in \mathbb{R}.$$

In what follows the symbol $\Longrightarrow$ denotes convergence in distribution.

Consider independent not necessarily identically distributed random variables $Y_1, Y_2, ...$ with $\mathsf{E}Y_i = 0$ and $0 < \sigma_i^2 = \mathsf{D}Y_i < \infty$, $i \geqslant 1$. For $k \in \mathbb{N}$ denote

$$S_k = Y_1 + ... + Y_k, \quad \overline{S}_k = \max_{1\leqslant i\leqslant k} S_i, \quad \underline{S}_k = \min_{1\leqslant i\leqslant k} S_i,$$

$B_k^2 = \sigma_1^2 + ... + \sigma_k^2$. Assume that the random variables $Y_1, Y_2, ...$ satisfy the Lindeberg condition: for any $\tau > 0$

$$\lim_{k\to\infty} \frac{1}{B_k^2}\sum\nolimits_{i=1}^k \int_{|x|\geqslant \tau B_k} x^2 d\mathsf{P}(Y_i < x) = 0. \tag{4}$$

It is well known that under the above conditions we have

$$\mathsf{P}\left(\frac{\overline{S}_k}{B_k} < x\right) \Longrightarrow \Psi(x), \quad \mathsf{P}\left(\frac{\underline{S}_k}{B_k} < x\right) \Longrightarrow 1 - \Psi(-x), \quad k \to \infty,$$

Let $N_1, N_2, \ldots$ be nonnegative integer-valued random variables such that for each $k \in \mathbb{N}$ the random variables $N_k, Y_1, Y_2, \ldots$ are independent. For $k \in \mathbb{N}$ set

$$S_{N_k} = Y_1 + \ldots + Y_{N_k}, \qquad \overline{S}_{N_k} = \max_{1 \leqslant i \leqslant N_k} S_i, \qquad \underline{S}_{N_k} = \min_{1 \leqslant i \leqslant N_k} S_i$$

(for definiteness we assume that $S_0 = \overline{S}_0 = \underline{S}_0 = 0$). Let $\{d_k\}_{k \geqslant 1}$ be an infinitely increasing sequence of positive numbers.

It is easy to see that the distribution function $H_\gamma(x)$ is absolutely continuous (the corresponding density has the form

$$h_\gamma(x) = \frac{1}{8}\int_0^\infty z^2 \exp\big\{-\tfrac{1}{8}xz^2\big\} g_{\gamma,1}(z)dz, \quad x \geqslant 0).$$

THEOREM 3. *Let W_γ and $\widetilde{W}_\gamma$ be random variables having respectively, Weibull distribution* (1) *with shape parameter $\gamma \in (0,1]$ and symmetric two-sided Weibull distribution* (3) *with the same parameter. Let $H_\gamma(x)$ be the distribution function defined in* (2). *Assume that Lindeberg condition* (4) *holds and $N_k \longrightarrow \infty$ in probability as $k \to \infty$. Then, as $k \to \infty$, the following statements are equivalent*:

$$\frac{S_{N_k}}{d_k} \Longrightarrow \widetilde{W}_\gamma; \qquad \frac{\overline{S}_{N_k}}{d_k} \Longrightarrow W_\gamma; \qquad \frac{\underline{S}_{N_k}}{d_k} \Longrightarrow -W_\gamma; \qquad \frac{|S_{N_k}|}{d_k} \Longrightarrow W_\gamma;$$

$$\sup_x \left|\mathsf{P}\Big(\frac{B^2_{N_k}}{d_k^2} < x\Big) - H_\gamma(x)\right| \longrightarrow 0.$$

We also prove a criterion of convergence of the distributions of statistics constructed from samples with random sizes to the symmetric two-sided Weibull distribution.

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A functional limit theorem for order flow imbalance processes

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In [1, 2] we use compound Cox processes to model order flows on financial exchanges taking into account stochastic nature of its intensities. We consider a time interval $[0;T]$ which is short enough so that the average parameters of the distributions of sizes of incoming orders could be assumed known within this interval. We consider well known *order imbalance process* ([3]) in the form

$$Q(t) = \sum_{i=1}^{N_1^+(\alpha^+\Lambda^*(t))} X_i^+ - \sum_{j=1}^{N_1^-(\alpha^-\Lambda^*(t))} X_i^-,$$

where X_i^+ are identically distributed sizes of buy orders, X_i^- are identically distributed sizes of sell orders, $N_1^+(\alpha^+\Lambda^*(t))$ and $N_1^-(\alpha^-\Lambda^*(t))$ are the counting processes for arrival of buy and sell orders and depending both on the process $\Lambda^*(t)$, the random measure playing role of external informational background, α^+ and α^-, the reaction degree of buyers and sellers to this information (and assumed constant within $[0;T]$).

We show that process $Q(t)$ is equal to process of the form

$$\sum_{j=1}^{N_1(\Lambda(t))} X_j,$$

where $\Lambda(t) = (\alpha^+ + \alpha^-)\Lambda^*(t)$ and X_j have a common characteristic function. For simplicity we put $T = 1$.

In order to introduce reasonable asymptotics which formalizes the condition of “infinite” growth of intensities of order flow, consider a sequence of compound Cox processes of the form

$$Q_n(t) = \sum_{i=1}^{N_1^{(n)}(\Lambda_n(t))} X_{n,i}, \quad t \geqslant 0, \tag{1}$$

where $\{N_1^{(n)}(t),\, t \geqslant 0\}_{n\geqslant 1}$ is a sequence of Poisson processes with unit intensities; for each $n = 1, 2, ...$ the random variables $X_{n,1}, X_{n,2}, ...$ are identically distributed; for any $n \geqslant 1$ the random variables $X_{n,1}, X_{n,2}, ...$ and the process $N_1^{(n)}(t)$, $t \geqslant 0$, are independent; for each $n = 1, 2, ...$ $\Lambda_n(t)$, $t \geqslant 0$, is a subordinator, that is, a non-decreasing positive Lévy process, independent of the process

$$Z_n(t) = \sum_{i=1}^{N_1^{(n)}(t)} X_{n,i}, \quad t \geqslant 0, \tag{2}$$

and such that $\Lambda_n(0) = 0$ and there exist $\delta \in (0, 1]$, $\delta_1 \in (0, 1]$ and the constants $C_n \in (0, \infty)$ providing for all $t \in (0, 1]$ the validity of the inequality

$$\mathsf{E}\Lambda_n^{\delta}(t) \leqslant (C_n t)^{\delta_1}. \tag{3}$$

Also assume that
$$\mathsf{P}\big(\Lambda_n(1) < k_n x\big) \xrightarrow{d} \mathsf{P}(U < x), \tag{4}$$
where U is a nonnegative random variable such that its distribution is not degenerate in zero.

Denote $a_n = \mathsf{E}X_{n,1}$ and assume that
$$0 < m_n^\beta \equiv \mathsf{E}|X_{n,1}|^\beta < \infty \text{ for some } \beta \in [1,2] \tag{5}$$
and for some $k_n \in \mathbb{N}$ the convergence
$$\mathsf{P}(X_{n,1} + ... + X_{n,k_n} < x) \xrightarrow{d} H(x) \tag{6}$$
takes place, where $H(x)$ is some infinitely divisible distribution function.

Theorem 1. *Let the compound Cox processes $Q_n(t)$ (see (1)) be lead by non-decreasing positive Lévy processes $\Lambda_n(t)$ satisfying conditions* (3) *and* (4) *with some $\delta, \delta_1 \in (0,1]$ and $k_n \in \mathbb{N}$. Assume that the random variables $\{X_{n,j}\}_{j\geqslant 1}$ satisfy conditions* (5) *with the same k_n and* (6) *with some $\beta \in [1,2]$. Also assume that condition*
$$K \equiv \sup_n C_n^{\delta_1/\delta} m_n^\beta < \infty$$
holds. Then order flow imbalance processes $Q_n(t)$ weakly converge in the Skorokhod space $\mathcal{D}$ to the Lévy process $Q(t)$ such that
$$\mathsf{E}\exp\{isQ(1)\} = \int_0^\infty \big(h(s)\big)^u \, d\mathsf{P}(U < u), \quad s \in \mathbb{R}, \tag{7}$$
where $h(s)$ is the characteristic function corresponding to the distribution function $H(x)$ in (6).

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On convergence of the distributions of statistics constructed from samples with random sizes to normal variance-mean mixtures

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Random sequences with independent random indexes play an important role in modeling real processes in many fields. Most popular examples of the application of these models usually deal with insurance and reliability theory, financial mathematics and queuing theory, chaotic processes in plasma physics where random sums are principal mathematical models. More general randomly indexed random sequences arrive in the statistics of samples with random sizes. Indeed, very often the data to be analyzed is collected or registered during a certain period of time and the flow of informative events each of which brings a next observation forms a random point process, so that the number of available observations is unknown till the end of the process of their registration and also must be treated as a (random) observation.

The literature on random sequences with random indexes is extensive. The mathematical theory of random sequences with random indexes is well-developed. However, there still remain some unsolved problems. For example, convenient conditions for the convergence of the distributions of general statistics constructed from samples with random sizes to normal variance-mean mixtures have not been found yet. At the same time, normal variance-mean mixtures are widely used as mathematical models of statistical regularities in many fields. In particular, in 1977–78 O. Barndorff-Nielsen [1, 2] introduced the class of *generalized hyperbolic distributions* as a class of special univariate variance-mean mixtures of normal laws in which the mixing is carried out in one parameter since location and scale parameters of the mixed normal distribution are directly linked. The range of applications of generalized hyperbolic distributions varies from the theory of turbulence or particle size description to financial mathematics, see [3]. Multivariate generalized hyperbolic distributions were introduced in the seminal paper [1] mentioned above as a natural generalization of the univariate case. They were further investigated in [4] and [5]. It is a convention to explain such a good adequacy of generalized hyperbolic models by that they possess many parameters to be suitably adjusted. But actually, it would be considerably more reasonable to explain this phenomenon by limit theorems yielding the possibility of the use of generalized hyperbolic distributions as convenient *asymptotic* approximations.

Let $m \in \mathbb{N}$. The vectors $\boldsymbol{x} = (x^{(1)}, \ldots, x^{(m)})^\top$ are elements of $\mathbb{R}^m$, the superscript $^\top$ stands for the transpose of a vector or matrix. The scalar product in $\mathbb{R}^m$ will be denoted $\langle\cdot,\,\cdot\rangle$: $\langle \boldsymbol{x},\,\boldsymbol{y}\rangle = \boldsymbol{x}^\top\boldsymbol{y} = x^{(1)}y^{(1)}+\ldots+x^{(m)}y^{(m)}$. As usual, the Euclidean norm of $\boldsymbol{x}$ is $\|\boldsymbol{x}\| = \langle \boldsymbol{x},\,\boldsymbol{x}\rangle^{1/2}$. If A is a real-valued $(m\times m)$-square matrix, then $\det(A)$ denotes the determinant of A. The $(m\times m)$-identity matrix is denoted $\boldsymbol{I}$. To properly distinguish between the real number zero and the zero vector, we write $0\in\mathbb{R}$ and $\mathbf{0} = (0,\ldots,0)^\top\in\mathbb{R}^m$. The notation $N_{\boldsymbol{a},\Sigma}$ will be used for the m-dimensional normal distribution with mean vector $\boldsymbol{a}$ and covariance matrix Σ. The distribution function of the one-dimensional standard normal distribution will be denoted $\Phi(x)$,

$$\Phi(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-y^2/2}dy, \quad x\in\mathbb{R}.$$

Assume that all the random variables and vectors considered below are defined on one and the same probability space $(\Omega,\,\mathfrak{F},\,\mathsf{P})$. The symbols $\mathfrak{B}_m$ and $\mathfrak{B}_+$ will denote the Borel sigma-algebras of subsets of $\mathbb{R}_m$ and $\mathbb{R}_+\equiv[0,\infty)$, respectively. In what follows the symbols $\stackrel{d}{=}$ and $\Longrightarrow$ will denote coincidence of distributions and weak convergence (convergence in distribution). We will write $\mathcal{L}(\boldsymbol{X})$ to denote the distribution of a random vector $\boldsymbol{X}$. A family $\{\boldsymbol{X}_j\}_{j\in\mathbb{N}}$ of $\mathbb{R}^m$-valued random vectors is said to be *weakly relatively compact*, if each sequence of its elements contains a weakly convergent subsequence. As is known, in the finite-dimensional case the weak relative compactness of a family $\{\boldsymbol{X}_j\}_{j\in\mathbb{N}}$ is equivalent to its *tightness* $\lim_{R\to\infty}\sup_{n\in\mathbb{N}}\mathsf{P}(\|\boldsymbol{X}_n\|>R)=0$.

Let $\{\boldsymbol{S}_{n,k} = (S_{n,k}^{(1)},\ldots,S_{n,k}^{(m)})^\top\}$, $n,k\in\mathbb{N}$, be a double array of $\mathbb{R}^m$-valued random vectors. For $n,k\in\mathbb{N}$ let $\boldsymbol{a}_{n,k} = (a_{n,k}^{(1)},\ldots,a_{n,k}^{(m)})^\top\in\mathbb{R}^m$ be non-random vectors and $b_{n,k}\in\mathbb{R}$ be real numbers such that $b_{n,k}>0$. The purpose of the vectors $\boldsymbol{a}_{n,k}$ and numbers $b_{n,k}$ is to provide weak relative compactness of the family of the random vectors $\big\{\boldsymbol{Y}_{n,k}\equiv b_{n,k}^{-1}\big(\boldsymbol{S}_{n,k}-\boldsymbol{a}_{n,k}\big)\big\}_{n,k\in\mathbb{N}}$ in the cases where it is required.

Consider a family $\{N_n\}_{n\in\mathbb{N}}$ of nonnegative integer random variables such that for each $n,k\in\mathbb{N}$ the random variables N_n and random vectors $\boldsymbol{S}_{n,k}$ are independent. Especially note that we do not assume the row-wise independence of $\{\boldsymbol{S}_{n,k}\}_{k\geqslant1}$. Let $\boldsymbol{c}_n = (c_n^{(1)},\ldots,c_n^{(m)})^\top\in\mathbb{R}^m$ be non-random vectors and d_n be real numbers, $n\in\mathbb{N}$, such that $d_n>0$. Our aim is to study the asymptotic behavior of the random vectors $\boldsymbol{Z}_n\equiv d_n^{-1}\big(\boldsymbol{S}_{n,N_n}-\boldsymbol{c}_n\big)$ as $n\to\infty$ and find rather simple conditions under which the limit laws for $\boldsymbol{Z}_n$ have the form of normal variance-mean mixtures. In order to do so we first formulate a somewhat more general result following the lines of [6], removing superfluous assumptions, relaxing the conditions and generalizing the results of that paper.

The characteristic functions of the random vectors $\boldsymbol{Y}_{n,k}$ and $\boldsymbol{Z}_n$ will be denoted $h_{n,k}(\boldsymbol{t})$ and $f_n(\boldsymbol{t})$, respectively, $\boldsymbol{t}\in\mathbb{R}^m$. Let $\boldsymbol{Y}$ be an $\mathbb{R}^m$-valued random vector whose characteristic function will be denoted $h(\boldsymbol{t})$, $\boldsymbol{t}\in\mathbb{R}^m$. Introduce the random variables $U_n = d_n^{-1}b_{n,N_n}$. Let $\boldsymbol{V}_n = (V_n^{(1)},\ldots,V_n^{(m)})^\top$ where $V_n^{(k)} = d_n^{-1}(a_{n,N_n}^{(k)}-c_n^{(k)})$ is the kth component of the random vector

$d_n^{-1}(\boldsymbol{a}_{n,N_n} - \boldsymbol{c}_n)$. In what follows by $\boldsymbol{W}_n$ we will denote the $(m+1)$-dimensional compound random vector $\boldsymbol{W}_n = (U_n,\, \boldsymbol{V}_n^\top)^\top = (U_n,\, V_n^{(1)}, \ldots, V_n^{(m)})^\top$.

Consider the function

$$g_n(\boldsymbol{t}) \equiv \mathsf{E} h(U_n \boldsymbol{t}) e^{i\langle \boldsymbol{t}, \boldsymbol{V}_n\rangle} = \sum_{k=1}^{\infty} e^{i\langle \boldsymbol{t}, d_n^{-1}(\boldsymbol{a}_{n,k} - \boldsymbol{c}_n)\rangle} h\big(d_n^{-1} b_{n,k} \boldsymbol{t}\big), \quad \boldsymbol{t} \in \mathbb{R}^m. \quad (1)$$

It can be easily seen that $g_n(\boldsymbol{t})$ is the characteristic function of the random vector $U_n \cdot \boldsymbol{Y} + \boldsymbol{V}_n$ where the random vector $\boldsymbol{Y}$ is independent of the random vector $\boldsymbol{W}_n$.

In the double-array limit setting considered in this paper, to obtain nontrivial limit laws for $\boldsymbol{Z}_n$ we require the following additional *coherency condition*: for any $T \in (0, \infty)$

$$\lim_{n\to\infty} \mathsf{E} \sup_{\|\boldsymbol{t}\| \leqslant T} \big|h_{n,N_n}(\boldsymbol{t}) - h(\boldsymbol{t})\big| = 0. \quad (2)$$

Remark 1. It can be easily verified that, since the values under the expectation sign in (2) are nonnegative and bounded (by two), then coherency condition (2) is equivalent to that $\sup_{\|\boldsymbol{t}\| \leqslant T} \big|h_{n,N_n}(\boldsymbol{t}) - h(\boldsymbol{t})\big| \longrightarrow 0$ in probability as $n \to \infty$.

Lemma 1. *Let the family of random variables $\{U_n\}_{n\in\mathbb{N}}$ be weakly relatively compact. Assume that coherency condition (2) holds. Then for any $\boldsymbol{t} \in \mathbb{R}^m$ we have*

$$\lim_{n\to\infty} |f_n(\boldsymbol{t}) - g_n(\boldsymbol{t})| = 0.$$

Lemma 1 makes it possible to use the distribution defined by the characteristic function $g_n(\boldsymbol{t})$ (see (1)) as an *accompanying asymptotic* approximation to the distribution of the random vector $\boldsymbol{Z}_n$. In order to obtain a *limit* approximation, we formulate and prove the following transfer theorem.

Theorem 1. *Assume that coherency condition (2) holds. If there exist a random variable U and an m-dimensional random vector $\boldsymbol{V}$ such that the distributions of the $(m+1)$-dimensional random vectors $\boldsymbol{W}_n$ converge to that of the random vector $\boldsymbol{W} = (U,\ \boldsymbol{V}^\top)^\top$:*

$$\boldsymbol{W}_n \Longrightarrow \boldsymbol{W} \quad (n \to \infty), \quad (3)$$

then

$$\boldsymbol{Z}_n \Longrightarrow \boldsymbol{Z} \stackrel{d}{=} U \cdot \boldsymbol{Y} + \boldsymbol{V} \quad (n \to \infty). \quad (4)$$

where the random vectors $\boldsymbol{Y}$ and $\boldsymbol{W} = (U,\ \boldsymbol{V}^\top)^\top$ are independent.

It is easy to see that relation (4) is equivalent to that the limit law for normalized randomly indexed random vectors $\boldsymbol{Z}_n$ is a scale-location mixture of the distributions which are limiting for normalized non-randomly indexed random vectors $\boldsymbol{Y}_{n,k}$. Among all scale-location mixtures, *variance-mean mixtures* attract a special interest. To be more precise, we should speak of *normal variance-mean mixtures* which are defined in the following way.

An $\mathbb{R}^m$-valued random vector $\boldsymbol{X}$ is said to have a multivariate normal mean-variance mixture distribution if $\boldsymbol{X} \stackrel{d}{=} \boldsymbol{a} + U\boldsymbol{b} + \sqrt{U}A\boldsymbol{Y}$, where $\boldsymbol{a}, \boldsymbol{b} \in \mathbb{R}^m$, A is a real $(m \times m)$-matrix such that the matrix $\Sigma \equiv AA^\top$ is positive definite, $\boldsymbol{Y}$ is a random vector with the standard normal distribution $N_{0,\boldsymbol{I}}$ and U is a real-valued, non-negative random variable independent of $\boldsymbol{Y}$. Equivalently, a probability measure F on $(\mathbb{R}^m, \mathfrak{B}_m)$ is said to be a multivariate normal mean-variance mixture if

$$F(d\boldsymbol{x}) = \int_0^\infty N_{\boldsymbol{b}+z\boldsymbol{a},\, z\Sigma}(d\boldsymbol{x})G(dz),$$

where the mixing distribution G is a probability measure on $(\mathbb{R}_+, \mathfrak{B}_+)$. In this case we will sometimes write $F = N_{\boldsymbol{b}+z\boldsymbol{a},\, z\Sigma} \circ G$.

Let us see how these mixtures can appear in the double-array setting under consideration. Assume that the centering vectors $\boldsymbol{a}_{n,k}$ and $\boldsymbol{c}_n$ are in some sense proportional to the scaling constants $b_{n,k}$ and d_n. Namely, assume that there exist vectors $\boldsymbol{a}_n \in \mathbb{R}^m$ and $\boldsymbol{b}_n \in \mathbb{R}^m$ such that for all $n, k \in \mathbb{N}$ we have $\boldsymbol{a}_{n,k} = d_n^{-1} b_{n,k}^2 \boldsymbol{a}_n$, $\boldsymbol{c}_n = d_n \boldsymbol{b}_n$, and there exist finite limits $\boldsymbol{a} = \lim_{n\to\infty} \boldsymbol{a}_n$, $\boldsymbol{b} = \lim_{n\to\infty} \boldsymbol{b}_n$. Then under condition (3) $\boldsymbol{W}_n = \big(U_n, (U_n^2\boldsymbol{a}_n + \boldsymbol{b}_n)^\top\big)^\top \Longrightarrow \big(U, (U^2\boldsymbol{a} + \boldsymbol{b})^\top\big)^\top$ $(n \to \infty)$, so that if in theorem 2 $\boldsymbol{Y}$ has the m-dimensional normal distribution $N_{\boldsymbol{0},\,\Sigma}$, then the limit law for $\boldsymbol{Z}_n$ takes the form of the normal variance-mean mixture $N_{\boldsymbol{b}+z\boldsymbol{a},\, z\Sigma} \circ G$ with G being the distribution of U^2.

In order to prove a result that is a partial inversion of theorem 1, for fixed random vectors $\boldsymbol{Z}$ and $\boldsymbol{Y}$ with the characteristic functions $f(\boldsymbol{t})$ and $h(\boldsymbol{t})$ introduce the set $\mathcal{W}(\boldsymbol{Z}|\,\boldsymbol{Y})$ containing all $(m+1)$-dimensional random vectors $\boldsymbol{W} = (U,\ \boldsymbol{V}^\top)^\top$ with $U \in \mathbb{R}$ and $\boldsymbol{V} \in \mathbb{R}^m$ such that the characteristic function $f(\boldsymbol{t})$ can be represented as

$$f(\boldsymbol{t}) = \mathsf{E}h(U\boldsymbol{t})e^{i\langle \boldsymbol{t},\boldsymbol{V}\rangle}, \quad \boldsymbol{t} \in \mathbb{R}^m, \tag{5}$$

and $\mathsf{P}(U \geqslant 0) = 1$. Whatever random vectors $\boldsymbol{Z}$ and $\boldsymbol{Y}$ are, the set $\mathcal{W}(\boldsymbol{Z}|\,\boldsymbol{Y})$ is always nonempty since it trivially contains the vector $(0, \boldsymbol{Z}^\top)^\top$. It is easy to see that representation (5) is equivalent to that $\boldsymbol{Z} \stackrel{d}{=} U\boldsymbol{Y} + \boldsymbol{V}$.

The set $\mathcal{W}(\boldsymbol{Z}|\,\boldsymbol{Y})$ may contain more that one element. For example, if $\boldsymbol{Y}$ is the random vector with standard normal distribution $N_{\boldsymbol{0},\boldsymbol{I}}$ and $\boldsymbol{Z} \stackrel{d}{=} \boldsymbol{T}_1 - \boldsymbol{T}_2$ where $\boldsymbol{T}_1$ and $\boldsymbol{T}_2$ are independent random vectors with independent components having the same standard exponential distribution, then along with the vector $\big(0, (\boldsymbol{T}_1 - \boldsymbol{T}_2)^\top\big)^\top$ the set $\mathcal{W}(\boldsymbol{Z}|\,\boldsymbol{Y})$ contains the vector $\big(\sqrt{U}, \boldsymbol{0}^\top\big)^\top$ where U is a random variable with the standard exponential distribution. In this case $\boldsymbol{Z}$ has the spherically symmetric Laplace distribution.

Let $\Lambda(\boldsymbol{X}_1,\ \boldsymbol{X}_2)$ be any probability metric which metrizes weak convergence in the space of $(m+1)$-dimensional random vectors. An example of such a metric is the Lévy–Prokhorov metric (see,e. g., [7]).

THEOREM 2. *Let the family of random variables* $\{U_n\}_{n\in\mathbb{N}}$ *be weakly relatively compact. Assume that coherency condition* (2) *holds. Then a random vector* $\boldsymbol{Z}$ *such that*

$$\boldsymbol{Z}_n \Longrightarrow \boldsymbol{Z} \quad (n \to \infty)$$

with some $\boldsymbol{c}_n \in \mathbb{R}^m$ *exists if and only if there exists a weakly relatively compact sequence of random vectors* $\boldsymbol{W}_n^* \equiv (U_n^*, (\boldsymbol{V}_n^*)^\top)^\top \in \mathcal{W}(\boldsymbol{Z}|\,\boldsymbol{Y})$, $n \in \mathbb{N}$, *such that*

$$\lim_{n\to\infty} \Lambda(\boldsymbol{W}_n^*,\ \boldsymbol{W}_n) = 0.$$

REMARK 2. It should be noted that in [6] and some subsequent papers a stronger and less convenient version of the coherency condition was used. Furthermore, in [6] and the subsequent papers the statements analogous to lemma 1 and theorems 1 and 2 were proved under the additional assumption of the weak relative compactness of the family $\{\boldsymbol{Y}_{n,k}\}_{n,k\in\mathbb{N}}$.

Let $\{\boldsymbol{X}_{n,j}\}_{j\geqslant 1}$, $n \in \mathbb{N}$, be a double array of row-wise independent not necessarily identically distributed random vectors with values in $\mathbb{R}^r$, $r \in \mathbb{N}$. For $n, k \in \mathbb{N}$ let $\boldsymbol{T}_{n,k} = \boldsymbol{T}_{n,k}(\boldsymbol{X}_{n,1}, \dots, \boldsymbol{X}_{n,k})$ be a statistic, i.e., a measurable function of $\boldsymbol{X}_{n,1}, \dots, \boldsymbol{X}_{n,k}$ with values in $\mathbb{R}^m$. For each $n \geq 1$ we define a random vector $\boldsymbol{T}_{n,N_n}$ by setting $\boldsymbol{T}_{n,N_n}(\omega) \equiv \boldsymbol{T}_{n,N_n(\omega)}(\boldsymbol{X}_{n,1}(\omega), \dots, \boldsymbol{X}_{n,N_n(\omega)}(\omega))$, $\omega \in \Omega$.

Let θ_n be $\mathbb{R}^m$-valued vectors, $n \in \mathbb{N}$. In this section we will assume that the random vectors $\boldsymbol{S}_{n,k}$ have the form $\boldsymbol{S}_{n,k} = \boldsymbol{T}_{n,k} - \theta_n$, $n, k \in \mathbb{N}$. Concerning the normalizing constants and vectors we will assume that there exist m-dimensional vectors $\boldsymbol{a}$, $\boldsymbol{a}_n$, $\boldsymbol{b}$, $\boldsymbol{b}_n$ and positive numbers σ_n such that

$$\boldsymbol{a}_n \to \boldsymbol{a}, \quad \boldsymbol{b}_n \to \boldsymbol{b} \quad (n \to \infty) \tag{6}$$

and for all $n, k \in \mathbb{N}$

$$b_{n,k} = (\sigma_n\sqrt{k})^{-1},\ d_n = (\sigma_n\sqrt{n})^{-1},\ \boldsymbol{a}_{n,k} = (\sigma_n k)^{-1}\sqrt{n}\boldsymbol{a}_n,\ \boldsymbol{c}_n = (\sigma_n\sqrt{n})^{-1}\boldsymbol{b}_n \tag{7}$$

so that

$$\boldsymbol{Y}_{n,k} = \sigma_n\sqrt{k}(\boldsymbol{T}_{n,k} - \theta_n) - \sqrt{n/k}\,\boldsymbol{a}_n \quad \text{and} \quad \boldsymbol{Z}_n = \sigma_n\sqrt{n}(\boldsymbol{T}_{n,N_n} - \theta_n) - \boldsymbol{b}_n.$$

As this is so, $\sigma_n^2\boldsymbol{I}$ can be regarded as the asymptotic variance of $\boldsymbol{T}_{n,k}$ as $k \to \infty$ whereas the bias of $\boldsymbol{T}_{n,k}$ is $\sqrt{n}(k\sigma_n)^{-1}\boldsymbol{a}_n$.

Recall that the characteristic function of the normal distribution in $\mathbb{R}^m$ with zero expectation and covariance matrix Σ is $\varphi(\boldsymbol{t}) = \exp\{-\frac{1}{2}\boldsymbol{t}^\top\Sigma\boldsymbol{t}\}$, $\boldsymbol{t} \in \mathbb{R}^m$. In what follows we will assume that the statistic $\boldsymbol{T}_{n,k}$ is asymptotically normal in the following sense: there exists a positive definite symmetric matrix Σ such that for any $T \in (0, \infty)$

$$\lim_{n\to\infty} \mathsf{E} \sup_{\|\boldsymbol{t}\|\leqslant T} \left|h_{n,N_n}(\boldsymbol{t}) - \exp\{-\tfrac{1}{2}\boldsymbol{t}^\top\Sigma\boldsymbol{t}\}\right| = 0, \tag{8}$$

where $h_{n,k}(\boldsymbol{t})$ is the characteristic function of the random vector $\boldsymbol{Y}_{n,k}$.

THEOREM 3. *Let the family of random variables* $\{n/N_n\}_{n\in\mathbb{N}}$ *be weakly relatively compact, the normalizing constants have the form* (7) *and satisfy condition* (6). *Assume that the statistic* $\boldsymbol{T}_{n,k}$ *is asymptotically normal so that condition* (8) *holds. Then a random vector* $\boldsymbol{Z}$ *such that*

$$\sigma_n\sqrt{n}(\boldsymbol{T}_{n,N_n}-\theta_n)-\boldsymbol{b}_n\Longrightarrow Z\quad(n\to\infty)$$

exists if and only if there exists a distribution function G *such that* $G(0)=0$, *the distribution* F *of* $\boldsymbol{Z}$ *has the form* $F=N_{\boldsymbol{b}+z\boldsymbol{a},\,z\Sigma}\circ G$ *and*

$$\mathsf{P}(n/N_n<x)\Longrightarrow G(x)\quad(n\to\infty).$$

REMARK 3. In limit theorems of probability theory and mathematical statistics, centering and normalization of random variables and vectors are used to obtain non-trivial asymptotic distributions. It should be especially noted that to obtain reasonable approximation to the distribution of the basic random variables (in our case, $\boldsymbol{T}_{n,N_n}$), both centering and normalizing values should be non-random. Otherwise the approximate distribution becomes random itself and, say, the problem of evaluation of quantiles becomes senseless.

The class of normal variance-mean mixtures is very wide. For example, it contains generalized hyperbolic laws with generalized inverse Gaussian mixing distributions, in particular, (a) symmetric and non-symmetric (skew) Student distributions (including Cauchy distribution), to which there correspond inverse gamma mixing distributions; (b) variance gamma (VG) distributions) (including symmetric and non-symmetric Laplace distributions), to which there correspond gamma mixing distributions; (c) normal\\inverse Gaussian (NIG) distributions to which there correspond inverse Gaussian mixing distributions, and many other types. Along with generalized hyperbolic laws, the class of normal variance-mean mixtures contains symmetric strictly stable laws with strictly stable mixing distributions concentrated on the positive half-line, generalized exponential power distributions and many other types.

Generalized hyperbolic distributions demonstrate exceptionally high adequacy when they are used to describe statistical regularities in the behavior of characteristics of various complex open systems, in particular, turbulent systems and financial markets. There are dozens of dozens of publications dealing with models based on univariate and multivariate generalized hyperbolic distributions. Therefore below we will concentrate our attention on limit theorems establishing the convergence of the distributions of statistics constructed from samples with random sizes to multivariate generalized hyperbolic distributions.

In order to do so, recall the definition of the *generalized inverse Gaussian distribution* $GIG_{\nu,\mu,\lambda}$ on $\mathfrak{B}_+$. The density of this distribution is denoted $p_{GIG}(x;\nu,\mu,\lambda)$ and has the form

$$p_{GIG}(x;\nu,\mu,\lambda)=\frac{\lambda^{\nu/2}}{2\mu^{\nu/2}K_\nu\big(\sqrt{\mu\lambda}\big)}\cdot x^{\nu-1}\cdot\exp\Big\{-\frac{1}{2}\Big(\frac{\mu}{x}+\lambda x\Big)\Big\},\quad x>0.$$

Here $\nu \in \mathbb{R}$,

$$\begin{aligned} &\mu > 0, \quad \lambda \geqslant 0, \quad \text{if } \nu < 0,\\ &\mu > 0, \quad \lambda > 0, \quad \text{if } \nu = 0,\\ &\mu \geqslant 0, \quad \lambda > 0, \quad \text{if } \nu > 0, \end{aligned}$$

$K_\nu(z)$ is the modified Bessel function of the third kind with index ν,

$$K_\nu(z) = \frac{1}{2}\int_0^\infty y^{\nu-1} \exp\Big\{ -\frac{z}{2}\Big(y + \frac{1}{y}\Big)\Big\} dy, \qquad z \in \mathbb{C},\ \operatorname{Re} z > 0.$$

The class of generalized inverse Gaussian distributions is rather rich and contains, in particular, both distributions with exponentially decreasing tails (gamma-distribution ($\mu = 0$, $\nu > 0$)), and distributions whose tails demonstrate power-type behavior (inverse gamma-distribution ($\lambda = 0$, $\nu < 0$), inverse Gaussian distribution ($\nu = -\frac{1}{2}$) and its limit case as $\lambda \to 0$, the Lévy distribution (stable distribution with the characteristic exponent equal to $\frac{1}{2}$ and concentrated on the nonnegative half-line, the distribution of the time for the standard Wiener process to hit the unit level)).

In the final part of his seminal paper [1], O. Barndorff-Nielsen defined the class of multivariate *generalized hyperbolic distributions* as the class of special normal variance-mean mixtures. Namely, let Σ be a positive definite $(m \times m)$-matrix with $\det(\Sigma)=1$, $\boldsymbol{a}$ and $\boldsymbol{b}$ be m-dimensional vectors. Then the m-dimensional generalized hyperbolic distribution $GH_{\nu,\mu,\lambda,\boldsymbol{a},\boldsymbol{b},\Sigma}$ on $\mathfrak{B}_m$ is defined as

$$GH_{\nu,\mu,\alpha,\boldsymbol{a},\boldsymbol{b},\Sigma} = N_{\boldsymbol{b}+z\Sigma\boldsymbol{a},\, z\Sigma} \circ GIG(\nu, \mu, \sqrt{\alpha^2 - \langle \boldsymbol{a}, \Sigma \boldsymbol{a}\rangle}).$$

Due to the restrictions imposed on the parameters of the generalized inverse Gaussian distribution, the parameters of generalized hyperbolic distribution must fit the conditions $\nu \in \mathbb{R}$, $\alpha, \mu \in \mathbb{R}_+$ and

$$\begin{aligned} &\mu > 0, \quad 0 \leqslant \langle \boldsymbol{a}, \Sigma\boldsymbol{a}\rangle \leqslant \alpha^2, \quad \text{if } \nu < 0,\\ &\mu > 0, \quad 0 \leqslant \langle \boldsymbol{a}, \Sigma\boldsymbol{a}\rangle < \alpha^2, \quad \text{if } \nu = 0,\\ &\mu \geqslant 0, \quad 0 \leqslant \langle \boldsymbol{a}, \Sigma\boldsymbol{a}\rangle < \alpha^2, \quad \text{if } \nu > 0, \end{aligned}$$

The corresponding distribution density $p_{GH}(\boldsymbol{x}; \nu, \mu, \alpha, \boldsymbol{a}, \boldsymbol{b}, \Sigma)$ has the form

$$p_{GH}(\boldsymbol{x}; \nu, \mu, \alpha, \boldsymbol{a}, \boldsymbol{b}, \Sigma) =$$

$$= \frac{(\alpha^2 - \langle \boldsymbol{a}, \Sigma\boldsymbol{a}\rangle)^{\nu/2}}{(2\pi)^{m/2}\alpha^{\nu-m/2}\mu^{\nu/2} K_\nu\big(\sqrt{\mu(\alpha^2 - \langle \boldsymbol{a}, \Sigma\boldsymbol{a}\rangle)}\big)} \sqrt{(\langle \boldsymbol{x} - \boldsymbol{b}, \Sigma^{-1}(\boldsymbol{x} - \boldsymbol{b})\rangle + \mu)^{\nu-m/2}} \times$$

$$\times K_{\nu-m/2}\big(\alpha\sqrt{\langle \boldsymbol{x} - \boldsymbol{b}, \Sigma^{-1}(\boldsymbol{x} - \boldsymbol{b})\rangle + \mu}\big) \exp\{\langle \boldsymbol{a}, \boldsymbol{x} - \boldsymbol{b}\rangle\}, \qquad \boldsymbol{x} \in \mathbb{R}^m.$$

THEOREM 4. *Let the family of random variables* $\{n/N_n\}_{n\in\mathbb{N}}$ *be weakly relatively compact, the normalizing constants have the form* (7) *and satisfy condition* (6) *with some* $\boldsymbol{a}, \boldsymbol{b} \in \mathbb{R}^m$. *Assume that the statistic* $\boldsymbol{T}_{n,k}$ *is asymptotically* *normal so that condition* (8) *holds with some symmetric positive definite matrix* Σ*. Then the distribution of a statistic* $\boldsymbol{T}_{n,N_n}$ *constructed from the sample with random size* N_n *weakly converges, as* $n \to \infty$*, to an* m*-dimensional generalized hyperbolic distribution*:

$$\mathcal{L}\big(\sigma_n\sqrt{n}(\boldsymbol{T}_{n,N_n} - \theta_n) - \boldsymbol{b}_n\big) \Longrightarrow GH_{\nu,\mu,\alpha,\Sigma^{-1}\boldsymbol{a},\boldsymbol{b},\Sigma}$$

if and only if

$$\mathcal{L}\big(n^{-1}N_n\big) \Longrightarrow GIG_{-\nu,\lambda,\mu} \tag{9}$$

with $\lambda = \sqrt{\alpha^2 - \langle \boldsymbol{a}, \Sigma\boldsymbol{a}\rangle}$.

This theorem is a straightforward corollary of theorem 3 with the account of a simply verifiable fact that if $\mathcal{L}(\xi) = GIG_{\nu,\mu,\lambda}$, then $\mathcal{L}(\xi^{-1}) = GIG_{-\nu,\lambda,\mu}$.

Theorem 4 can serve as convenient explanation of the high adequacy of generalized hyperbolic Lévy distributions as models of statistical regularities in the behavior of stochastic systems. Moreover, they directly link the mixing distribution in the representation of a generalized hyperbolic distribution with the random sample size which is determined by the intensity of the flow of informative events generating the observations.

According to theorem 4, for example, to obtain the limit multivariate asymmetric Student distribution for $\boldsymbol{T}_{n,N_n}$ it is necessary and sufficient that in (9) the mixing distribution is the gamma distribution. To obtain the multivariate variance gamma limit distribution for $\boldsymbol{T}_{n,N_n}$ it is necessary and sufficient that in in (24) the mixing distribution is the inverse gamma distribution. In particular, for $\boldsymbol{T}_{n,N_n}$ to have the limit multivariate asymmetric Laplace distribution it is necessary and sufficient that the limit distribution for $n^{-1}N_n$ is inverse exponential.

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Dilemmas of robust analysis of economic data streams

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Data streams (streaming data) consist of transiently observed, temporally evolving multidimensional data sequences that challenge our computational and/or inferential capabilities. In the Economics, data streams are among others related to fraud detection in retail banking (credit card transactions), financial markets or electricity consumption monitoring, public eye or social networks monitoring, and the Internet users behaviours exploring. Analysis of the economic data streams introduces several new challenges to the statistical analysis involving need for online processing and online inference, and temporal adaptivity of our decision schemes in the face of unforeseen changes, both smooth and abrupt, in the underlying data generation mechanism.

Due to existence of outliers in the economic data sets, robust statistical procedures are used more and more often. Unfortunately, a great part of good robust statistical procedures are computationally and/or memory very intensive. Due to certain substantive conceptual issues related to a notion of an influential majority of the data – a great part of good robust statistical procedures do not allow for their recursive formulation in a similar manner as in cases of the mean vector, the covariance matrix or the least squares regression. Due to our pour knowledge of general laws ruling economic phenomena – an usage of well known Kalman filter machinery is computationally infeasible.

In this paper we study possibilities of overcoming these substantial computational difficulties related to robust analysis of the economic data stream. We introduce models for the economic data streams basing on well known models for multiregime time series with random as well as deterministic switching. Then we discuss several strategies for reducing complexity of robust analysis of the data stream. The considered strategies involve using so called micro-clusters, robust binning of the data and using representative objects for the systems basing on inspection of their trajectories. Within the paper we discuss advantages and disadvantages of usage of statistical tools offered by the so called data depth concept (i.e.,e.g., local depths, depths for functional data).

Data depth concept was originally introduced as a way to generalize the concepts of median and quantiles to the multivariate framework. A depth function $D(\cdot, F)$ associates with any $\mathbf{x} \in \mathbb{R}^d$ a measure $D(\mathbf{x}, F) \in [0, 1]$ of its centrality w.r.t. a probability measure $F \in \mathcal{P}$ over $\mathbb{R}^d$ or w.r.t. an empirical measure $F_n \in \mathcal{P}$ calculated from a sample $\mathbf{X}^n$. The larger the depth of $\mathbf{x}$, the more central $\mathbf{x}$ is w.r.t. to F or F_n. The most celebrated examples of the depth known in the literature are Tukey and Liu depth. For our purposes, the most interesting depth seems to be the weighted L^p depth. The weighted L^p depth $WL^pD(\mathbf{x}; F)$ of a point $\mathbf{x} \in \mathbb{R}^d, d \geqslant 1$ being a realization of some d dimensional random vector $\mathbf{X}$ with distribution F, is defined as

$$WL^pD(\mathbf{x}; F) = \frac{1}{1 + Ew(\|\mathbf{x} - \mathbf{X}\|_p)},$$

where E denotes the expectation, w is a suitable weight function on $[0, \infty)$, and $\|\cdot\|_p$ stands for the L^p norm. Fig. 1 presents the sample contour plot for the L^2 depth.

We discuss possibilities of recursive and/or distributed formulation of selected robust multivariate statistical procedures and show their properties using very big financial data sets as well as simulation studies.

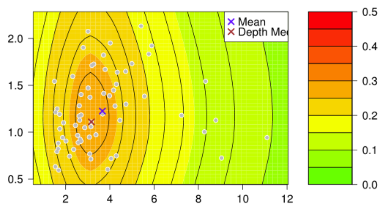

Figure 1: Sample L^2 depth contour plot (DepthProc package).

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Problems in calculating of the moments and the distribution function of the ladder height

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The problem of approximate calculation of the moments and the distribution function of the ladder height is considered. Algorithms are proposed for calculating the moments by the formulas from [5], including the algorithm for finding solutions of the Frobenius equation [8]. Chebyshev's method is applied to restore the distribution function via continued fractions.

Metod of moments. Chebyshev concluded an expression of approximately value for distribution function in context known moments of a random variable in [1-3]. It includes functions of continued fractions, the explicit formulas of it are presented in [6] and expressed in terms of the moments.

The moments of the ladder height. In [5] an expression of calculation of moments of the ladder height Z_+ is concluded by Fa di Bruno's formula [8]. Under a condition a step has a distribution $N(0, \sigma^2)$, $\sigma = 1$, $m > 0$, it looks like

$$\mathsf{E}Z_+^{m+1} = \frac{(m+1)!}{\sqrt{2}} \sum_{\{jk_j\}_1^m} \prod_{j=1}^{m} \left(\frac{1}{k_j!} \left(\frac{g_j}{j!} \right)^{k_j} \right),$$

where explicit form of values g_i can be deduced according [7].

$$g_i = -(i-1)! \frac{\cos \frac{\pi i}{4}}{2^{i-1} i \pi^{i/2}} \sum_{n=1}^{\infty} \frac{1}{n^{i/2}}, \quad i \geqslant 3;$$

$$g_1 = \frac{K}{\sqrt{2\pi}}, \quad K : \sum_{m=1}^{n} \frac{1}{\sqrt{m}} = 2\sqrt{n} - K + O(\frac{1}{\sqrt{n}}); \quad g_2 = \frac{1}{4}.$$

Calculations. In [4] Sonin has shown that the inaccuracy of Chebyshev metod decreases like $1/n$ in case of normal distributed random value, where n is number of known moments. The calculations for the ladder height confirm it if $n = 18, 26$. Thus, necessity of calculating at least 100 moments is obvious. Besides, this metod is very susceptible to precision of calculations. Used algorythm can't be realized even by super-computer, if $n > 28$. The other algorythm is in process now.

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On multi-channel networks approximation by the Ornstein-Uhlenbeck process

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The main model in question is a stochastic $[M_t|M|\infty]^r$-network consisting of r service nodes. From the outside a non-homogeneous Poisson flow of calls $\nu_i(t)$ with the leading function $\Lambda_i(t)$, $i = 1, 2, ..., r$, arrives at the i-th node. Each of these "r" nodes operates as a multi-channel stochastic system. If the call arrives at such a system then its service immediately begins. The service time in the i-th node is exponentially distributed with parameter μ_i, $i = 1, 2, ..., r$. After completion of service in the i -th node the call arrives to the j-th node with probability p_{ij} and leaves the network with probability

$p_{ir+1} = 1 - \sum_{j=1}^{r} p_{ij}$. Let us note $P = \|p_{ij}\|_1^r$ as the switching matrix of the network. An additional node numbered "$r+1$" is interpreted as "output" from the network.

We will define the service process in the network as an r-dimensional process $Q(t) = (Q_1(t), ..., Q_r(t))'$, where $Q_i(t)$ is the number of calls in the i-th node at the moment of time t. Our main purpose is to study the conditions under which the process $Q(t)$ may be approximated by the r-dimentional Ornstein-Uhlenbeck process.

We will assume that characteristics of the $[M_t|M|\infty]^r$-network depend on a series parameter n in such a way:

Condition 1. $\lim_{n\to\infty} n\mu_i^{(n)} = \mu_i > 0$, $i = 1, 2, ..., r$.

Condition 2. For any $T > 0$, we have

$$\sup_{t\in[0,T]} \left| n^{-1}\Lambda^{(n)}(nt) - \lambda t \right| = o(n^{-1/2}),$$

where $\lambda = (\lambda_1, \lambda_2, ..., \lambda_r)'$, $\lambda_i \geqslant 0$, $i = 1, 2, ..., r$ and $\lambda_1 + \lambda_2 + ... + \lambda_r \neq 0$.

Clearly, if the Condition 2 is hold then random flows $\nu_i(t)$, $i = 1, 2, ..., r$, in the time-scale nt are close to the stationary Poisson flow with the parameter λ_i. Taking into account both Conditions 1 and 2 means that $[M_t|M|\infty]^r$-network operates in a heavy traffic regime.

For the initial state of the network provided that it is open (spectral radius of the matrix P is strictly less than 1) we will demand the implementation of the following condition:

Condition 3. $Q_i^{(n)}(0) = \left[n\theta_i/\mu_i + \sqrt{n}\xi_i^{(0)} \right], i = 1, 2, ..., r,$

where $\theta' = (\theta_1, ..., \theta_r) = \lambda'(I - P)^{-1}$, $I = \|\delta_{ij}\|_1^r$ is the identity matrix, $\xi^{(0)} = (\xi_1^{(0)}, ..., \xi_r^{(0)})' \in R^r$ is a fixed vector and [.] is the integer part.

Now we are ready to present the main result of the work.

Theorem 1. *Let for the $[M_t|M|\infty]^r$- network with the spectral radius strictly less than 1 the Conditions 1-3 be hold. Then for any finite interval $[0, T]$ the sequence of stochastic processes*

$$\xi^{(n)}(t) = n^{-1/2} \left(Q^{(n)}(nt) - n(\theta/\mu) \right), (\theta/\mu)' = (\theta_1/\mu_1, ..., \theta_r/\mu_r),$$

converges, in the uniform topology, to the Ornstein-Uhlenbeck diffusion $\xi^{(0)}(t)$ ($\xi^{(0)}(0) = \xi^{(0)}$) with a drift vector $A(x) = (P' - I)\Delta(\mu)x$ and a diffusion matrix $B = \Delta(\theta)(P - I) + (P' - I)\Delta(\theta)$, where $\Delta(z) = \|z_i\delta_{ij}\|_1^r$ is a diagonal matrix with the vector $z' = (z_1, ..., z_r)$ on the principal diagonal.

The proof is based on the method developed in the work [1].

In closing we will consider networks with variable parameters of input flows $\lambda_i(t)$ ($\Lambda_i(t) = \int_0^t \lambda_i(u)du$), $i = 1, 2, ..., r$, that are periodically varied: $\lambda_i(nT_i + u) = \lambda_i(u)$, for $n = 1, 2, ...,$ $0 \leqslant u \leqslant T_i$. It is not difficult to show that for such models the Condition 2 is hold under $\lambda_i(t) = \int_0^{T_i} \lambda_i(u)du$, $i =$

$1, 2, ..., r$, and therefore we can use the theorem 1 to construct the Ornstein-Uhlenbeck approximate process.

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Forest fire on configuration random graphs

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The study of random graphs' robustness to different types of breakdowns has been one of the important trends in the field of random graphs (see e.g. Durret [1], Norros and Reittu [2]). Here we consider random graphs' resilience from the viewpoint of node survival. This aspect branched off the studies of forest fire propagation models (see e.g. Bertoin [3], Drossel and Schwabl [4]), as well as modeling of banking system defaults (Arinaminparty et al. [5]).

We consider configuration random graphs (see Durret [1], Hofstad [6]) of N nodes numbered from 1 to N with node degrees $\xi_1, \xi_2, \ldots, \xi_N$ drawn independently from a given distribution. This distribution defines the number of enumerated stubs for each node. The graph is constructed by joining all the stubs pairwise equiprobably to form links. In order to form all links one stub is added to a random node if the sum of node degrees is odd. We consider two types of node degree distributions leading to two graph types: power-law and Poisson random graphs with parameters $\tau > 1$ and $\lambda > 0$, respectively.

We view graph nodes as trees on a confined area of a real forest placed in the vertices of a square lattice sized 100×100. Links connect nodes in a closest neighbour manner. The link exists if a fire can propagate between neighbouring nodes. Thus, in a fully packed lattice every inner node has 8 adjacent neighbours. An average node degree m is related to the parameters of power-law and Poisson node degree distributions through Riemann zeta function as $m = \zeta(\tau) = \lambda$. Therefore we consider graphs which sizes $N \leq 10000$ depend on node degree distribution parameters. Fire propagation starts from either a node with the highest degree (target fire start) or an equiprobably chosen node (random fire start) spreading to neighbouring nodes with an initially set probability $0 < p \leq 1$. The aim of the work is to find the best topology of configuration random graph that saves maximum of nodes in case of a fire.

We performed computer simulations of fire propagation for both graph types in two fire start cases: random and target. These simulations allowed us to find the optimal values of node degree distribution parameters τ and λ that ensure maximum survival of graph nodes as well as to derive regression relationships between the number of survivor nodes g, the node degree distribution parameter (τ or λ) and the probability of fire transition p. The results for power-law graph models are given in Leri and Pavlov [7].

Both power-law and Poisson graph models showed to be more resilient to random fire start than to targeted ignition. We also compared the number of survivor nodes under the same initial state conditions (values of N and p) for both graph types. In the case of a random fire start the power-law graph topology allows more trees to survive than the Poisson node degree distribution. However when a fire starts through lightning striking the tree with the highest number of links the topology that will give the highest node survival depends on both the fire transition probability p and the initial graph size N.

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On improper priors and conditional sampling

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It is well known that improper priors in Bayesian statistics may lead to proper posterior distributions and useful inference procedures. This motivates the presentation of an elementary theoretical frame for statistics that includes improper priors, consisting in a relaxation of Kolmogorov's axioms to allow infinite mass. The theory gives an alternative to common ad hoc arguments which are not based on an underlying theory, and it leads to simple explanations of apparent paradoxes described in the literature. The role of improper distributions in fiducial statistics and conditional sampling will be discussed in particular.

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On structure of periodically correlated sequences

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Let $T \geq 1$ be a fixed positive integer. A periodically correlated sequence of period T is a a sequence $x(n)$, $n \in Z$, in a Hilbert space such that its autocorrelation function $R(n,m) = (x(n), x(m))$ has the property that $R(n,m) = R(n+T, m+T)$, $n, m \in Z$. Probabilists may think about a sequence of second-order random variables with mean zero and the autocorrelation function defined as $R(n,m) = Ex(n)\overline{x(m)}$. A periodically correlated sequence with period $T = 1$ is called stationary. A basic fact in the theory of stationary sequences states that every stationary sequence is of the form $x(n) = U^n x$, where U^n is a unitary representation of the group of integers Z. It turns out that the structure of periodically correlated sequences involves an interplay of two unitary representations: a representation U^n of Z and a representation V^λ of the group $\Lambda = \{2\pi k/T : k = 0, 1, \ldots, T-1\}$ regarded as a subgroup of the torus $[0, 2\pi)$. To be more precise we will show that a sequence $(x(n))$ is periodically correlated with period T if and only if there are a Hilbert K (usually larger than the space spanned by $(x(n))$), a vector $x \in K$, and two unitary operators U and V in K, such that $V^T = I$, $VU = e^{-2\pi i/T} UV$, and

$$x(n) = (1/T) \sum_{j=0}^{T-1} e^{-2\pi i j n/T} U^n V^j x, \qquad n \in Z.$$

The triple (U, V, x) above is unique in the sense of unitary equivalence.

The theorem reveals a surprising relation between periodically correlated sequences and the canonical commutation relation. We will discuss some consequences of this theorem in both theory of periodically correlated sequences and in abstract harmonic analysis.

Thank you.

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Spectral analysis and modeling of non-Gaussian processes of structural plasma turbulence

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Within researches of the low-frequency plasma fluctuations lying in ranges of frequencies to 100 MHz, the status of strong structural low-frequency turbulence was revealed [1,2].This turbulence is described by mathematical model of non-uniform casual wandering with continuous time, namely twice stochastic Poisson process differently called by generalized Cox process [3].

For the purpose of definition of number of the processes forming structural turbulence, the analysis of increments of density of probabilities of low-frequency plasma fluctuations was carried out. The following stage of researches it was necessary to pass to the analysis of frequency ranges [4], as the range gives the chance to define instability type, the mechanism of formation of turbulence, the mechanism of its saturation to make quantitative estimates of structures (ion-sound solitons and drift vortices) etc. However the analysis of such ranges Fourier analysis traditional methods be impossible, for example, robast Fourier-spectrum of ion-sound structural turbulence can is approximated by different models [5]: from Kolmogorov-Obukhova model to shot noise. The main complexity of identification of stochastic processes of structural turbulence on a broadband range was that at known number of processes, the form of harmonics into which the peak range could be divided, remained the unknown.

We developed an empirical approach to the analysis of broadband ranges of the low-frequency structural plasma turbulence, based on aprioristic assumptions about number of processes, their scales (estimated of histograms) and a Gaussian form spectral a component. Steady ranges of low-frequency turbulence [6] are interpreted as density of some unknown probability distribution [7]. The program created for the bootstrap analysis of the harmonics, which implements the described algorithm. Such empirical approach allowed divide Fourier-spectra of low-frequency plasma turbulence into components. On fig.1 the complex range of the turbulence measured by Doppler reflectometry in stellarator L-2M is shown. In this range allocated three harmonics (over experimental noise) that have a characteristic Doppler frequency shifts. Shift of the main harmonic is connected with radial electric field, i.e. is defined by plasma poloidal rotation speed (or plasma fluctuations). Doppler shifts of other associated with phase velocities of two types of structural plasma turbulence.

In the report it is shown [8] that on a gradient of density of plasma in stellarator L-2M (r/a=0.9...0.95) can exist both electronic-temperature gradient (ETG), and ion-temperature gradient (ITG) instabilities. The linear dispersion equations for drift instabilities of both types are presented in [9]. On fig.2 dependencies increments on wave number for two instabilities that initiate corresponding structural turbulence are given. Phase velocities of fluctuations which result from development of these instabilities, are directed on electronic and ionic drift of particles in a magnetic field of a stellarator. As seen on fig. 1 in a range except a harmonic connected with poloidal rotation of plasma, allocated two more harmonics, which correspond to fluctuations of a rotating in opposite directions, which corresponds to the direction of the electron and ion drift. Such researches were carried out for three various modes of existence of plasma in L-2M with current heating and electronic-cyclotron heating (two capacities 200 and 400 kW). These modes are different conditions on the buildup of the instability and the phase velocity fluctuations. In all modes it was possible to allocate the components connected with poloidal rotation

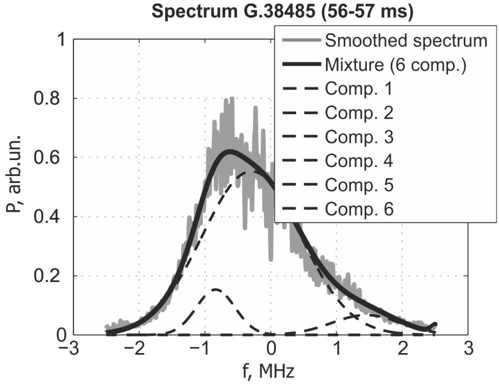

Figure 1: Robust turbulent spectrum of Doppler reflectometry diagnostics decomposed into 6 components. 3 components were over experimental noise. The solid line represents the average spectrum, the dotted lines show the three main spectral components.

of plasma (is defined by radial electric field), and phase velocity of structural turbulence of two types (are defined by instabilities of ETG and ITG).

The carried-out successful description of probabilistic and spectral characteristics of low-frequency plasma turbulence allowed to set a correct task about modeling of structural turbulence by system of the stochastic differential equations. These equations should consider casual processes with the density which has been set in the form of a final mix of probabilistic distributions. Such comprehensive approach will allow to carry out comparison of models of plasma processes (for example, drift dissipative and ion-sound instability processes, gradient instabilities, etc.) with characteristics of the received stochastic processes.

Acknowledgements. This work is executed with grant support by the Russian Federation Presidential Grant (No. MK-5607.2013.2).

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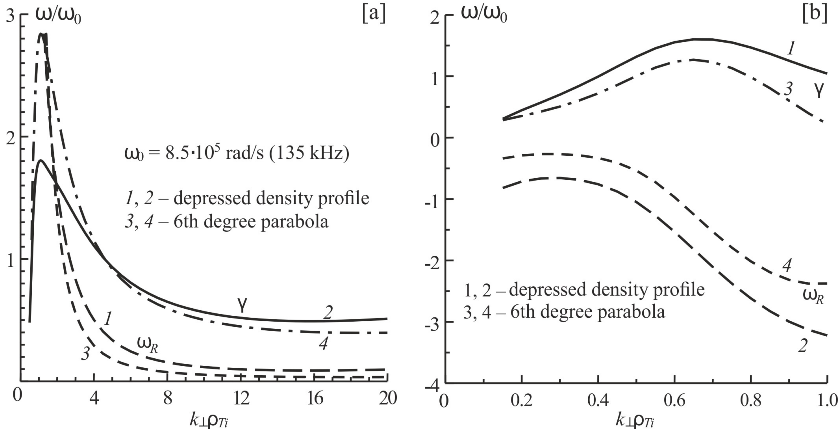

Figure 2: Dependences increments on wave number for ETG [a] and ITG [b] instabilities on plasma density gradient on the edge in stellarator L-2M.

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Verification statistical hypothesis about ES value in finite sample setting

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Introduced on the turn of 21st century, the axiomatic risk theory has developed around the notion of a coherent risk measure. In recent literature much attention has been given to the ES (expected shortfall) measure, which fulfils the set of coherency axioms and offers an important extension to the VaR model. The idea behind ES measure is to give information about the possible loss in case of extreme events. In case of countinuous real random variables, the definition of ES reduces to the expectation of the distribution tail.

The wide variety of ES-based risk models, introduced in the recent literature, created the need for relevant testing procedures. In the general case, the distribution of a sample average of extreme observations is unknown, thus classic statistical methods are unfeasible for ES value testing. Since scarcity of observations is inherent to extreme events, the statistical inference cannot be based on the central limit theorem, which requires large sample size.

Since the beginning of the 21st century several approaches have been proposed for ES model backtesting or ES value verification. The use of bootstrap technique V, which is based on the simulated distribution of the test statistic, was proposed by McNeil and Frey [2]. Its modification V^*, aimed at using more sample information, was suggested by Embrechts [1]. Finally, circumventing the problem of the unknown distribution, Wong [3] introduced the saddlepoint test technique S, which gives approximate p-values through the Taylor expansion of the moment generating function.

The aim of the paper was to evaluate statistical properties of available ES value testing procedures. Test assessment included their size and power. The analysis of the test properties was preceded by the overview of statistical inference methods proposed in the literature for ES models. The statistical properties of the considered tests were evaluated through the Monte Carlo method.

The size and power evaluation experiments were designed in a way that they reflected volatility clustering phenomenon, which hinders volatility prediction and is commonly regarded as a key issue in risk control. Volatility clustering was represented through inclusion of a GARCH process in the data generating algorithm. The size estimates for the considered tests are given in Table 1.

The power evaluation was based on three variants of the simulation experiment. We used GARCH models with undersized standard deviations, fixed at chosen percent of the true standard deviation: $0.9\sigma_t$, $0.7\sigma_t$ and $0.5\sigma_t$, where σ_t denotes the correct parameter value. The power evaluation results are presented in Table 2.

The results showed that type one errors for the saddlepoint test S and the bootstrap test V, assuming series length of at least 250 data, were compliant with the assumed significance level of 5%. The power comparison showed that for the sample size of 250 observations the highest rejection frequencies under the alternative were observed for the V test. The saddlepoint test S rejection frequencies were slightly lower, however there was a clear growth in the power estimates with lengthening the time series.

Test	Series length			
	250	500	750	1000
S	0.047	0.054	0.049	0.052
V	0.056	0.053	0.055	0.052
V^*	0.140	0.164	0.162	0.172

Table 1: Size estimates of ES tests

Test	σ_t^*	Series length			
		250	500	750	1000
S	$0,9\sigma_t$	0.34	0.51	0.64	0.71
	$0,7\sigma_t$	0.59	0.70	0.82	0.88
	$0,5\sigma_t$	0.89	0.97	0.99	1.00
V	$0,9\sigma_t$	0.49	0.50	0.56	0.62
	$0,7\sigma_t$	0,55	0,74	0,85	0,91
	$0,5\sigma_t$	0,87	0,97	0,99	1,00
$V*$	$0,9\sigma_t$	0.26	0.31	0.37	0.38
	$0,7\sigma_t$	0.54	0.64	0.73	0.80
	$0,5\sigma_t$	0.92	0.98	1.00	1.00

Table 2: Power estimates of ES tests

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Optimum estimators for the modified Weibull distribution of censored data

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The accuracy of various estimation techniques has been studied extensively for the Weibull distribution, which is commonly used for the lifetime data [3]. Imposing strong restrictions on the data, the Weibull model is unable to fit data that exhibit a bathtub-shaped hazard-rate function. Thus, basing on a real data on final products from the large production company, we suggested the use of the modified Weibull distribution, proposed by Lai, Xie and Murthy [2], to represent the shape of the failure rate function.

The survival function of the modified Weibull distribution is given as

$$S(t) = \exp\big(-at^b\exp(\lambda t)\big), \tag{1}$$

where $a > 0,\ b \geq 0,\ \lambda > 0$. The density and the hazard rate functions have the following forms:

$$f(t) = -S'(t) = a(b+\lambda t)t^{b-1}\exp(\lambda t)\exp\big(-at^b\exp(\lambda t)\big), \tag{2}$$

$$h(t) = a(b+\lambda t)t^{b-1}\exp(\lambda t). \tag{3}$$

This distribution can describe both increasing ($b \geq 1$) and bathtub-shaped ($0 < b < 1$) hazard functions and includes the Weibull distribution and the type I extreme value distribution as special cases [4]. Moreover, having three parameters it offers important numerical advantage over other more flexible distributions, which often have four or more parameters. Fig. 1. shows the observed number of failures and the shape of the density of the modified Weibull distribution estimated from the real data sample.

Five techniques proposed it the literature for censored data modelling have been compared in terms of their capability to estimate the parameters of the modified Weibull distribution. As benchmark methods we used the popular maximum likelihood and least squares estimators. Ross [5] and Jacquelin [1]

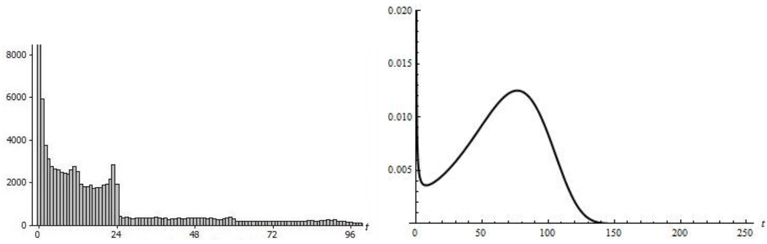

Figure 1: Observed number of failures and estimated density of the modified Weibull distribution.

estimator techniques were used as unbiasing factors for the maximum likelihood estimates. Finally we considered the White estimator [6], which is a weighted version of the least squares technique.

The study concentrated on II-type censored data. Estimator accuracy was evaluated through the Monte Carlo method. The bias of the expected values and variance of the parameter estimators were computed over 10000 simulations. The study included the bias understood as the difference between the expected value of the estimator and the true value of the parameter, as well as the fractional bias, which is the ratio of the expected value and the true parameter value. The focus was on the independence of the fractional bias of the parameter value. The research was conducted for sample sizes of n = 10, 20, 50, 100, 1000. The share of censored observations was set to 30, 60 and 90%.

The presented study allowed for recommendations about optimum estimators of the modified Weibull distribution in terms of feasibility and complexity of the techniques as well as their accuracy. All considered methods resulted in parameter estimates, which had a systematic error. The results showed that the commonly used maximum likelihood and least squares techniques are not to be recommended on censored data sets.

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Confidence intervals for average success probabilities

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We provide Buehler-optimal one-sided and some valid two-sided confidence intervals for the average success probability of a possibly inhomogeneous fixed length Bernoulli chain, based on the number of observed successes. Contrary to some claims in the literature, the one-sided Clopper-Pearson intervals for the homogeneous case are not completely robust here, not even if applied to the special case of hypergeometric estimation problems.

To be more precise, let B_p for $p \in [0,1]$, $\mathrm{B}_{n,p}$ for $n \in \mathbb{N}_0$ and $p \in [0,1]$, and $\mathrm{BC}_p := \ast_{j=1}^{n} \mathrm{B}_{p_j}$ for $n \in \mathbb{N}_0$ and $p \in [0,1]^n$ denote the Bernoulli, binomial, and Bernoulli convolution (or Poisson-binomial) laws with the indicated parameters. Then, for $n \in \mathbb{N}$ and $\beta \in \,]0,1[$, and writing $\overline{p} := \frac{1}{n}\sum_{j=1}^{n} p_j$ for $p \in [0,1]^n$, we are interested in β-confidence regions for the estimation problem

$$\left(\left(\mathrm{BC}_p \colon p \in [0,1]^n\right), [0,1]^n \ni p \mapsto \overline{p}\right), \tag{1}$$

that is, in functions $\mathrm{K}\colon \{0,\ldots,n\} \to 2^{[0,1]}$ satisfying $\mathrm{BC}_p\left(\mathrm{K} \ni \overline{p}\right) \geqslant \beta$ for $p \in [0,1]^n$. Clearly, every such K is also a β-confidence region for the binomial estimation problem

$$\big((\mathrm{B}_{n,p} \colon p \in [0,1]), \mathrm{id}_{[0,1]}\big), \tag{2}$$

that is, satisfies $\mathrm{B}_{n,p}\left(\mathrm{K} \ni p\right) \geqslant \beta$ for $p \in [0,1]$, but, as noted in [1] and thus refuting claims in several later publications such as [2], the converse is false for example if K is the Clopper-Pearson β-confidence upray for (2), namely

$$\mathrm{K}(x) \;=\; \begin{cases} [0,1] & \text{if } x = 0, \\ ]g_n(x), 1] & \text{if } x \in \{1,\ldots,n\}, \end{cases} \tag{3}$$

where $g_n(x) :=$ the $p \in [0,1]$ with $\mathrm{B}_{n,p}(\{x,\ldots,n\}) = 1-\beta$. We prove:

Theorem. *Let* $\beta \in [\frac{3}{4}, 1[$. *Then*

$$\mathrm{K}(x) \ := \ \begin{cases} [0,1] & \text{if } x = 0, \\ ]\frac{1-\beta}{n}, 1] & \text{if } x = 1, \\ ]g_n(x), 1] & \text{if } x \in \{2, \dots, n\} \end{cases} \tag{4}$$

defines the optimal isotone β-confidence upray for (1), is admissible in the set of all β-confidence uprays for (1), is strictly isotone, and has the effective level $\inf_{p\in[0,1]^n} \mathrm{BC}_p\,(\mathrm{K} \ni \overline{p}) = \beta$.

Thus (3) and (4) differ, but only in that $g_n(1) = 1 - \beta^{1/n} > (1-\beta)/n$.

The common source of the wrong claims in the literature indicated above is an unclear remark in [3]. On the other hand, our proof of the above theorem uses first a reduction, well known from [3] but essentially already presented in [4], to Bernoulli convolutions BC_p such that the coordinates of p take on at most one value different from 0 or 1, and then certain additional inequalities from [3] and [5].

We further prove for $\beta \geqslant \frac{1}{2}$ that the two-sided Clopper-Pearson β-confidence interval for (2) is a β-confidence interval for (1), but also that this robustness property does not extend to some other and less conservative competitors.

Definitions assumed above, somewhat more general forms of the results indicated, proofs, and further details can be found in [6].

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Comparative analysis of models of regulated and unregulated pedestrian crossing

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Stochastic models of transport systems have been attracting attention of many mathematicians during the last decade. The interest was driven by practical importance of these models for planning and organising of automobiles traffic as well as by complex purely mathematical problems arising in the area.

This work is closely linked to the papers in which mathematical models, describing unregulated intersection of two car roads, are built (see e.g.[1],[2]).

We consider a pedestrian crossing on a car road. Pedestrians approach the crossing in accordance to a Poisson process with intensity λ_1. The times required for the pedestrians to cross the road do not depend on the process of their arrivals to the crossing. These times are independent random variables with common distribution function B(x) where $b = \int_0^\infty x dB(x) < \infty$.

At first we consider the model where the pedestrians have absolute priority over the vehicles approaching the crossing. It means that the vehicle may proceed only when there are no pedestrians crossing the road. Otherwise the vehicle stops and waits until the road is free. After that the time required for the vehicle to complete the crossing is assumed to be a random variable with distribution function F(x). If there are no pedestrians and there is no queue of other cars at the moment of the vehicle approaching the pedestrian crossing, then the time required for it to complete the crossing is assumed to be equal to zero (the effect of skipping). The flow of the cars approaching the pedestrian crossing is assumed to be a Poisson process with intensity λ_2 .

One can conclude that the number of the pedestrians Q(t) at the moment $t \geq 0$ equals to the number of calls in a queuing system with infinite number of service elements: $M \mid GI \mid \infty$. It is well known that the limit distribution of Q(t) when $t \to \infty$ is the Poisson one with the intensity $\lambda_1 b$.

Another important characteristic of this system is its busy period, i.e. the period during which there is at least one pedestrian on the crossing. The Laplace- Stilties transformation of the distribution of the length of the busy period is defined by quite complex formula. We will consider only two partial cases: constant and exponential distribution of the service time.

We will model the number of vehicles queuing in front of the pedestrian crossing as a single-channel queuing system with unreliable service element. The element goes faulty when the first pedestrian emerges on the crossing. The element becomes operational at the end of the busy period of the system with infinite number of service channels described above. One can find a stationary distribution of the number of waiting vehicles and the moments of this distribution.

Intuitively it is obvious that if the intensity of the vehicles flow on the road is high, then the presence of a pedestrian crossing leads to a traffic jump, i.e. big queue consisted of vehicles. That is why it makes sense to include the traffic light into the model.

The traffic light works in the following way: for the vehicles the green light is present during the time τ_1, and the red light is present during the time τ_2, where τ_1, τ_2 are constants. For the pedestrians the reverse situation is true.

Here the line of vehicles again is described by single-channel queuing system with unreliable service element, which is operational during the time τ_1, and faulty during the time τ_2.

We propose an algorithm for estimation of major characteristics of the model, in particular, average number of the vehicles queuing in front of the traffic light.

The behaviour of the pedestrians is well described by queuing system with infinite number of service elements operating in random external environment. When the traffic light for the pedestrians goes red all the elements become faulty, when the traffic light goes green all the elements become operational. We assume that the service, interrupted because of the element goes faulty (the traffic light shows red colour), starts from the beginning when the element is operational again (the traffic light shows green colour).

Various approaches estimating the distribution of the number of queuing pedestrians are proposed. Some of them is proposed in [3]. Comparative analysis of the models with and without traffic light allows finding the boundaries for the intensities of the vehicles and pedestrians flows for which the installation of the traffic light is desirable.

The author expresses his deep gratitude to Professor L.G. Afanasyeva for formulation of the problem and for useful discussion.

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Levy processes and stochastic integrals with respect to generalized convolutions

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My talk is based on the paper *Lévy processes and stochastic integral in the sense of generalized convolution* written together with M. Borowiecka-Olszewska, B.H. Jasiulis-Gołdyn Rosiński. In this paper, we present a comprehensive theory of generalized and weak generalized convolutions, illustrate it by a large number of examples, and discuss the related infinitely divisible distributions. We consider Lévy and additive process with respect to generalized and weak generalized convolutions as certain Markov processes, and then study stochastic integrals with respect to such processes. We introduce the representability property of weak generalized convolutions. Under this property and the related weak summation, a stochastic integral with respect to random measures related to such convolutions is constructed.

Motivated by the seminal work of Kingman [2], K. Urbanik introduced and developed the theory of generalized convolutions in his fundamental papers starting from [5]. Roughly speaking, a generalized convolution is a binary associative operation $\star$ on probability measures such that the convolution of point-mass measures $\delta_x \star \delta_y$ can be a non-degenerate probability measure, while the usual convolution gives δ_{x+y}. The study of weakly stable distributions, initiated by Kucharczak and Urbanik and followed by a series of papers by Urbanik, Kucharczak, Panorska, and Vol'kovich, provided a new and rich class of weak generalized convolutions on $\mathbb{R}_+$ (called also $\mathcal{B}$-generalized convolutions). Misiewicz, Oleszkiewicz and Urbanik [3] gave full characterization of weakly stable distributions with non-trivial discrete part and proved some uniqueness properties of weakly stable distributions that will be used in this paper.

Examples of generalized convolutions.

0. The classical convolution is evidently an example of generalized convolution. It will be denoted as usual by $*$:

$$\delta_a * \delta_b = \delta_{a+b}.$$

1. *Symmetric* generalized convolution on $\mathcal{P}_+$ is defined by

$$\delta_a *_s \delta_b = \frac{1}{2}\,\delta_{|a-b|} + \frac{1}{2}\,\delta_{a+b}.$$

2. In a similar way another generalized convolution (called by Urbanik $(\alpha, 1)$-convolution in can be defined for every $\alpha > 0$ by

$$\delta_a *_{s,\alpha} \delta_b = \frac{1}{2}\,\delta_{|a^\alpha - b^\alpha|^{1/\alpha}} + \frac{1}{2}\,\delta_{(a^\alpha + b^\alpha)^{1/\alpha}}.$$

3. For every $p \in (0, \infty]$ the formula

$$\delta_a *_p \delta_b = \delta_c, \qquad a, b \geqslant 0, \;\; c = \|(a,b)\|_p = (a^p + b^p)^{1/p}$$

defines a generalized convolution $*_p$ (p-stable convolution) on $\mathcal{P}_+$.

4. The Kendall convolution $\diamond_\alpha$ on $\mathcal{P}_+$, $\alpha > 0$, is defined by

$$\delta_x \diamond_\alpha \delta_1 = x^\alpha \pi_{2\alpha} + (1 - x^\alpha)\delta_1, \quad x \in [0,1],$$

where $\pi_{2\alpha}$ is a Pareto measure with density $g_{2\alpha}(x) = 2\alpha x^{-2\alpha-1}\mathbf{1}_{[1,\infty)}(x)$.

5. The Kingman convolution $\otimes_{\omega_s}$ on $\mathcal{P}_+$, $s > -\frac{1}{2}$, is defined in [2] by

$$\delta_a \otimes_{\omega_s} \delta_b = \mathcal{L}\left(\sqrt{a^2 + b^2 + 2ab\theta_s}\right),$$

where θ_s is absolutely continuous with the density function

$$f_s(x) = \frac{\Gamma(s+1)}{\sqrt{\pi}\,\Gamma(s+\frac{1}{2})}\left(1 - x^2\right)_+^{s-\frac{1}{2}}.$$

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Consistency and asymptotic normality for kernel based nonparametric prediction under heterogeneous measurement errors

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Carroll, Delaigle and Hall [1] consider the problem of predicting the random variable Y nonparametrically via the estimation of $\mu(t) = E(Y|T = t)$. T is an observed "future" explanatory variable generated by $T = X + U^F$ where X is the true unobserved explanatory variable and U^F is a measurement error. The prediction problem is complicated by the fact that "past" observations $\{(Y_j, W_j)\}_{j=1}^n$ are such that $W_j = X_j + U_j$ with measurement errors U_j that are different from U^F. Moreover, the U_j themselves may have different distributions. They have suggested a new estimator for $\mu(t)$ and obtained its consistency.

Our paper provides two novel results in the context of this model. First, consistency of their estimator is provided under much less restrictive conditions. The main condition for consistency in their paper, and in the extant literature, is expressed in terms of the functional

$$v(h) = nh^{-1} \int \left|\varphi_K(t)\varphi_{f_{U^F}}\left(\frac{t}{h}\right)\right|^2 / \sum_{k=1}^n \left|\varphi_{f_{U_k}}\left(\frac{t}{h}\right)\right|^2 dt.$$

Here, K is a kernel, φ_K is its Fourier transform and φ_{f_X} is the characteristic function of a density f_X associated with a random variable X. If certain regularity assumptions are satisfied and $v(h)/n \to 0$, then the estimator proposed in [1] is consistent.

To obtain that $v(h)/n \to 0$ it is often required, as in [1], that φ_K have a compact support. Up to now, it has been unknown if $v(h)/n \to 0$ is possible when the support of φ_K is not compact and $\sum_{k=1}^n |\varphi_{f_{U_k}}(x)|^2$ declines at infinity exponentially fast. We provide a method to study the properties of $v(h)$. The method applies when $1/\sum_{k=1}^n |\varphi_{f_{U_k}}(x)|^2$ can be dominated by any of the iterated exponential functions $e_1(x) = \exp(x),\ e_2(x) = e_1(e_1(x)), ..., e_n(x) = e_1(e_{n-1}(x))$. Denoting

$$\Phi_n(s) = \frac{n\left|\varphi_{f_{U^F}}(s)\right|^2}{\sum_{k=1}^n \left|\varphi_{f_{U_k}}(s)\right|^2}.$$

we assume that

Assumption. Φ_n is locally bounded, that is $\sup_{s\in K}\Phi_n(s) < \infty$ for each compact $K \subset R$, and has a majorant P in the neighborhood of infinity such that

(a) with some positive c_1, c_2 one has $\Phi_n(s) \le c_2 P(s)$ for all $|s| \ge c_1$,

(b) P is even, $P(s) = P(-s)$, and with some $c_3 > 0$ the inequality $P(s) \le c_3 P'(s)$ holds for all $s \ge c_1$,

(c) $\int_{c_1}^{\infty} \exp(-P(s)) \left(1 + |P'(s)|^2\right) ds < \infty$.

(d) From (b) it follows that P is increasing on $[c_1, \infty)$, P^{-1} exists and is defined on $[P(c_1), \infty)$. Lastly, we require that

$$J(h) \equiv \int_{P(c_1)}^{\infty} \exp\left[-P(hP^{-1}(t))\right] dt < \infty \quad \text{for all } 0 < h < 1.$$

The inequalities $\Phi_n(s) \le c_2 P(s)$, $P(s) \le c_3 P'(s)$ can be replaced by their consequence $\Phi_n(s) \le cP'(s)$, still providing enough structure for our applications. We prefer to use the two inequalities for better transparency. Examples of functions P are $P(s) = \exp(s^\alpha)$ and iterated exponential functions. Iterated exponential functions form a scale that covers all imaginable errors. Note that $J(h)$ is monotone and therefore it is bounded from above when h is bounded away from zero. Assumption 1 has been developed with growing Φ_n in mind, because the case of a bounded Φ_n is simpler. Under this assumption we show that there exists a kernel $K \in L_1$, where the support of φ_K is not compact and $v(h) < \infty$ for all $0 < h < 1$. Furthermore, K satisfies $v(h)/n = o(1)$ with suitably chosen $h = h_n$.

The second novel result in our paper is the provision of sufficient conditions for the asymptotic normality of the estimator proposed in [1] when the measurement errors U_j are of two types and their characteristic functions, as well as that of U^F, are super-smooth. The weak convergence we obtain depends on a restriction on the class of K provided in van Es and Uh [2].

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The extension of the spectral method to the Harris Markov chains

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Let $\{X_n\}$ be the Markov chain defined on the measurable space $(X, \mathbf{S})$ with the transition function $p(x,B), x \in X, B \in \mathbf{S}$. Let $S_n = \sum\limits_{j=1}^{n} f(X_j)$, where $f(\cdot)$ is a real measurable function on $(X, \mathbf{S})$. In the theory of general Markov chains prevail direct probabilistic methods. The analytical approach is used only for uniformly ergodic chains. Our purpose is to extend the spectral method introduced in Nagaev [1] to the general case when the uniform ergodicity of the chain X_n is not supposed. Select some set $A_0 \in \mathbf{S}$. Let $v = \min\{n > 0 : X_n \in A_0\}, q_n(x,B) = \mathbf{P}\{v = n, X_n \in B | X_0 = x\}$, where $x \in A_0$. Define the new transition function on $(A_0, A_0\mathbf{S})$ by the equality $q(x,B) = \sum\limits_{n=1}^{\infty} q_n(x,B)$. We assume that the chain defined by this transition function is uniformly ergodic. Let $\mathfrak{M}$ and $\mathfrak{M}_0$ be the spaces of bounded complex functions respectively on $(X, \mathbf{S})$ and $(A_0, A_0\mathbf{S})$. Define the operators $P(t)$ and $P_1(t)$ by the formulas $P(t)g(x) = \int\limits_X g(y)e^{itf(y)}p(x,dy)$ and $P_1(t)g(x) = \int\limits_{A_0} g(y)e^{itf(y)}p(x,dy), g \in \mathfrak{M}$. Let $P_2(t) = P(t) - P_1(t)$. Denote $P_2^{k-1}(t)P_1(t)$ by $Q_k(t)$. Let $Q(z,t) = \sum\limits_{k=1}^{\infty} Q_k(t)z^k, |z| \leq 1$. The spectrum of $Q(1,0)$ has the isolated point 1. The rest of the spectrum is contained in the circle of the radius $\rho < 1$. According to the perturbation theory the spectrum of $Q(z,t)$ has the same structure for (z,t) close to $(1,0)$. The key formula is $P_0(z,t) = -R(1;z,t)$. Here $R(u;z,t)$ is the resolvent of $Q(z,t)$, and $P_0(z,t)$ is the contraction of $P(z,t) := \sum\limits_{n=0}^{\infty} P^n(t)z^n$ onto $\mathfrak{M}_0$. Hence $P_0^n(t) = -\frac{1}{2\pi i} \int\limits_{|z|=1} R(1;z,t)z^{-n-1}dz$. If (z,t) is close to $(1,0)$, then $R(1;z,t) = (1-\lambda(z,t))^{-1}Q_1(z,t) + \Omega(z,t)$, where the operator $\Omega(z,t)$ is uniformly bounded in some neighborhood of $(1,0)$ with respect to (z,t), and $Q_1(z,t)$ is the projector corresponding to the largest eigen-value $\lambda(z,t)$ of $Q(z,t)$. It follows from the conditions imposed on $q(x,\cdot)$ that $||\frac{\partial R}{\partial z}(1; e^{i\varphi}, t)||$ is uniformly bounded in $\{(\varphi,t) : \varepsilon < |\varphi| \leq \pi, |t| \leq \delta\})$ for every $0 < \varepsilon \leq \pi$ if δ is small enough. Hence, $\lim\limits_{n\to\infty} \int\limits_{\varepsilon<|\varphi|\leq\pi} e^{-ni\varphi}R(1;e^{i\varphi},t)d\varphi = 0$ uniformly in $|t| \leq \delta$. As a result we get

$$P_0^n(t) = -Q_1(1,0)(2\pi)^{-1} \int\limits_{|\varphi|\leq\varepsilon} (1-\lambda(e^{i\varphi},t))^{-1}d\varphi + T_n(t,) \qquad (1)$$

where $\lim_{n\to\infty} \sup_{|t|<\delta} ||T_n(t)|| = 0$. Further,

$$(2\pi)^{-1} \int\limits_{|\varphi|\le\varepsilon} e^{-in\varphi}(1-\lambda(e^{i\varphi},t))^{-1}d\varphi \sim$$

$$(2\pi)^{-1} \int\limits_{-\pi}^{\pi} e^{-in\varphi}(\lambda'_z(1,0)(1-e^{i\varphi}) - \lambda''_t(1,0)t^2/2)^{-1}d\varphi \sim$$

$$1/\lambda'_z(1,0)(1-\lambda''_t(1,0)t^2/2\lambda'_z(1,0))^n. \qquad (2)$$

It follows from (1) and (2) that

$$\lambda'_z(1,0) \lim_{n\to\infty} P_0^n\left(\frac{t}{\sqrt{n}}\right) = \exp\left\{\frac{\lambda''_t(1,0)}{2\lambda'_z(1,0)}t^2\right\}. \qquad (3)$$

Basing on (3)we prove that

$$\lim_{n\to\infty} P^n(t) = \exp\left\{\frac{\lambda''_t(1,0)}{2\lambda'_z(1,0)}t^2\right\}P_1, \qquad (4)$$

where $P_1 g(\cdot) \equiv \int\limits_X g(x)p_0(dx)$, $p_0(\cdot)$ being the stationary distribution for the chain X_n. On the other hand,

$$P^n(t)\mathbf{1}_X = \mathbf{E}\{e^{itS_n}|X_0\}, \qquad (5)$$

where $\mathbf{1}_X :=$ Ind X. It follows from (4) and (5) that $n^{-1/2}S_n$ is asymptotically normal $N(0, \sqrt{-\lambda''_t(1,0)/\lambda'_z(1,0)})$.

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On a non-uniform bound of the remainder term in central limit theorem for Bernoulli distributions

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Let Z, Z_1, Z_2, ... , Z_n be a sequence of independent Bernoulli random variables with the same distribution: $\mathbf{P}(Z = 1) = p$, $\mathbf{P}(Z = 0) = q = 1-p$. Denote the distribution functions of the normalized sum $\frac{1}{\sqrt{npq}} \sum\limits_{j=1}^{n} (Z_j - p)$ and the standard normal random variable by $F_n(x)$ and $\Phi(x)$ respectively. Introduce the following notations,

$$\delta_n(p,x) = F_n(x) - \Phi(x), \quad Q_1(x) = \frac{1-2p}{6\sqrt{pq}}\,(1-x^2), \quad \beta_3(p) = \mathbf{E}\Big|\frac{Z-p}{\sqrt{pq}}\Big|^3.$$

In the case when distribution functions are continuous from the left there exists a discontinuity point x_0 of the function $F_n(x)$, such that $\sup\limits_{x\in\mathbb{R}} |\delta_n(p,x)|$ is $\delta_n(p,x_0+)$ or $-\delta_n(p,x_0)$. For the sake of simplicity we discuss here only the case $\sup\limits_{x\in\mathbb{R}} |\delta_n(p,x)| = \delta_n(p,x_0+)$.

According to the result by C.-G. Esseen [1, p. 56] the following equality holds at the discontinuity points of $F_n(x)$ when $n \to \infty$,

$$\delta_n(p,x+) = \frac{1}{\sqrt{2\pi n}}\, e^{-x^2/2}\Big(Q_1(x) + \frac{1}{2\sqrt{pq}}\Big) + o\big(n^{-1/2}\big)$$

uniformly in x.

Other bound is found in the present work for the remainder term, which gives the opportunity to localize the maximum point of $\delta_n(p,x)$ in x in contrast to the Esseen result, namely, the following representation of $\delta_n(p,x+)$ holds for $n \geqslant 200$ and $p \geqslant 0.02$ at each discontinuity point of $F_n(x)$,

$$\frac{\sqrt{n}}{\beta_3(p)}\,\delta_n(p,x+) = \frac{\sqrt{n}}{\beta_3(p)}\left(\frac{1}{\sqrt{2\pi n}}\, e^{-x^2/2}\Big(Q_1(x) + \frac{1}{2\sqrt{pq}}\Big)\right) + R_1(p,n,x) + R_2(p,n,x),$$

where

$$|R_1(p,n,x)| \leqslant 0.012 + \frac{0.1\sqrt{n}}{\beta_3(p)}\,\exp\Big\{-\Big(\frac{\sqrt{n}}{\beta_3(p)}\Big)^2 1.64\Big\},$$

$$|R_2(p,n,x)| \leqslant e^{-x^2/2}(0.068\,|x| + 0.051|x^3 - 3x|).$$

This result together with the results of the paper [2] allows to reduce the time computing significantly when evaluating the Berry–Esseen absolute constant in the case of two-point distributions. In this case the problem is to find $\max\limits_{x,n,p} \frac{\sqrt{n}}{\beta_3(p)} |\delta_n(p,x)|$. If the explicit expression $F_n(x) = \sum\limits_{0\leqslant k<x\sqrt{npq}+np} \binom{n}{k} p^k q^{n-k}$ is used for computation, the result obtained by us gives opportunity to allocate quite a narrow region of the values of x, in which this maximum is attained.

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The convergence rate estimates for the generalized risk process with Pareto mixing

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Consider the doubly stochastic Poisson processes

$$S(t) = \sum_{k=0}^{N^*(t)} X_k,$$

where $N^*(t)$ is Mixed Poisson Pareto distributed r.v. with parameters α and δt, i.e. $N^*(t) \sim MPP(\alpha, \delta t)$.

$S(t)$ can represent the total claim amount process, so the corresponding claim sizes could be presented by i.i.d.r.v. $X_1, X_2, \ldots$ with common d.f. F

Let $X_1, X_2, \ldots$ satisfy the following moment conditions:

$$\mathsf{E}X_1 \equiv a, \quad \sigma^2 = \mathsf{E}X_1^2 < \infty.$$

We suppose that for each $t > 0$ the random variables $N^*(t), X_1, X_2, \ldots$ are independent.

We can interpret the $S(t)$ as the doubly stochastic Poisson processes controlled by $\Lambda(t)$ processes, where $\Lambda(t) = \Lambda_{\alpha,\delta} \cdot t$ and $\Lambda_{\alpha,\delta} \sim Pareto(\alpha, \delta)$.

In [Korolev (1996)] the necessary and sufficient conditions of the weak convergence of distributions of the doubly stochastic Poisson processes to the scale mixtures of normal laws with zero means were given.

Let $d(t) > 0$ is some auxiliary normalizing (scaling) unrestrictedly increasing as $t \to \infty$ function.

Auxiliary theorem. *Assume that $\Lambda(t) \to \infty$ as $t \to \infty$. Then the one-dimensional distributions of a normalized doubly stochastic Poisson processes $S(t)$ weakly converge to that of some random variable Z:*

$$\frac{S(t)}{\sigma\sqrt{d(t)}} \xrightarrow{d} Z \quad (t \to \infty),$$

if and only if there exists a nonnegative random variable Λ such that

1) $\mathsf{P}(Z < x) = \mathsf{E}\Phi\left(x/\sqrt{\Lambda}\right) = \int\limits_0^{+\infty} \Phi\left(x/\sqrt{\lambda}\right) d\mathsf{P}(\Lambda < \lambda), \quad x \in R,$

2) $\Lambda(t)/d(t) \xrightarrow{d} \Lambda \quad (t \to \infty)$.

In our case we have that $\mathsf{E}\Lambda(t)$ exist and equals

$$\mathsf{E}\Lambda(t) = \mathsf{E}\Lambda_{\alpha,\delta}t = t\mathsf{E}\Lambda_{\alpha,\delta} = \frac{\alpha\delta}{\alpha-1}t, \quad \alpha > 1.$$

This equality immediately allows us to take the function $d(t)$ normalizing a process $S(t)$ in Auxiliary theorem in the form $d(t) = t$.

So, if we do the normalization by the identity function $d(t) = t$ we will get the following limiting law Λ for $\Lambda(t)/t$:

$$\Lambda(t)/t \stackrel{d}{=} \Lambda_{\alpha,\delta} \equiv \Lambda.$$

Now we can get the explicit form of limiting low Z for normalized process $[S(t) - N^*(t)\mathsf{E}X_1]/(\sigma\sqrt{t})$ in terms of the generalized hypergeometric function

$$\mathsf{P}(Z < x) = \mathsf{E}\Phi\left(x/\sqrt{\Lambda}\right) = \int\limits_\delta^{+\infty} \Phi\left(x/\sqrt{\lambda}\right) d\mathsf{P}(\Lambda < \lambda) =$$

$$= \frac{1}{2} + \frac{2\alpha x}{\sqrt{2\pi\delta}(1+2\alpha)} \cdot {}_2F_2\left(\left[\frac{1}{2}, \frac{1}{2}+\alpha\right], \left[\frac{3}{2}, \frac{3}{2}+\alpha\right], -\frac{x^2}{2\delta}\right).$$

Lets estimate the accuracy of the approximation the distribution of $[S(t) - N^*(t)\mathsf{E}X_1]/(\sigma\sqrt{t})$ by the scale mixtures of normal law $\mathsf{E}\Phi\left(x/\sqrt{\Lambda}\right)$ found above. Denote

$$\Delta_t \equiv \sup_x \left| \mathsf{P}\left(\frac{S(t) - N^*(t)\mathsf{E}X_1}{\sigma\sqrt{t}} < x\right) - \right.$$

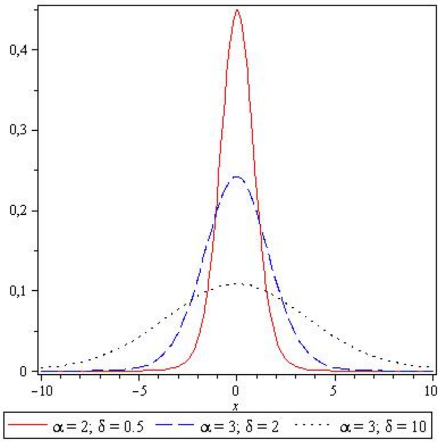

Figure 1: The plots of the density function $p_Z(x)$ of the limiting law Z for some α and δ.

$$-\frac{1}{2} - \frac{2\alpha x}{\sqrt{2\pi\delta}(1+2\alpha)} \cdot {}_2F_2\left(\left[\frac{1}{2}, \frac{1}{2}+\alpha\right], \left[\frac{3}{2}, \frac{3}{2}+\alpha\right], -\frac{x^2}{2\delta}\right)\Bigg|.$$

Theorem 1. *Assume that $\beta_3 \equiv \mathsf{E}|X_1|^3 < \infty$. Then for each $t > 0$ the following estimate is true*

$$\Delta_t \leq C(\alpha,\delta)\frac{\beta_3}{\sigma^3\sqrt{t}}, \quad \text{where} \quad C(\alpha,\delta) = \frac{0{,}3041\alpha}{(\alpha+1/2)\sqrt{\delta}}.$$

In particular, for parameters $\alpha = 2$ and $\delta = 0{,}5$ we get

$$\Delta_t \leq 0{,}3441\frac{\beta_3}{\sigma^3\sqrt{t}}.$$

Now we will consider the risk process

$$R(t) = c(t) - S(t), t \geq 0,$$

where $c(t)$ is the income curve.

It is intuitively clear that the intensity $\Lambda(t)$ of the flow of claims should be proportional to the portfolio size. So, it is naturally to assume that $c(t) = u + c\Lambda(t)$ and consider the following generalization of the risk process:

$$R_g(t) = u + c\Lambda(t) - S(t).$$

Here we will investigate the asymptotic behavior of such generalized risk process in the 'critical' case $c = a \equiv \mathsf{E}X_1$ and construct the convergence rate estimates in the central-limit-type theorem.

Using the special representation for $S(t)$ via $S_{N_\lambda} = \sum_{k=0}^{N_\lambda} X_k$ with classical Poisson process $N_\lambda,\ \lambda > 0$ and the analogue of the central limit theorem for S_{N_λ} :

$$\mathsf{P}\left(\frac{S_{N_\lambda} - \lambda \mathsf{E} X_1}{\sigma\sqrt{\lambda}} < x\right) \rightrightarrows \Phi(x), \quad \lambda \longrightarrow \infty,$$

we can easily formulate the central-limit-type theorem for risk process $R(t)$:

$$\frac{R_g(t)}{\sigma\sqrt{t}} \xrightarrow{d} Z, \quad t \longrightarrow \infty,$$

where

$$\mathsf{P}(Z < x) = \mathsf{E}\Phi\left(x/\sqrt{\Lambda_{\alpha,\delta}}\right) = \int\limits_0^{+\infty} \Phi\left(x/\sqrt{\lambda}\right) d\mathsf{P}(\Lambda_{\alpha,\delta} < \lambda), \quad x \in \mathbb{R}.$$

The explicit form of the limiting distribution of Z with $\Lambda_{\alpha,\delta} \sim Pareto(\alpha,\delta)$ was found above.

Lets estimate the accuracy of the approximation in this central-limit-type theorem for generalized risk process $R_g(t) = u + \mathsf{E}X_1\Lambda_{\alpha,\delta}t - S(t)$.

Theorem 2. *Assume that $\beta_3 \equiv \mathsf{E}|X_1|^3 < \infty$. Then for each $t > 0$ the following convergence rate estimate in the limit theorem for normalized generalized risk process $R_g(t)$ is true*

$$\sup_x \left| \mathsf{P}\left(\frac{R_g(t)}{\sigma\sqrt{t}} < x\right) - \mathsf{E}\Phi\left(x/\sqrt{\Lambda_{\alpha,\delta}}\right) \right| \leq$$

$$\leq \frac{1}{\sqrt{t}}\left(\frac{0.3041\beta_3}{\sigma^3} + \frac{u}{\sigma\sqrt{2\pi}}\right)\frac{\alpha}{(\alpha+1/2)\sqrt{\delta}}.$$

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On sampling plans for inspection by variables

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Consider the setting in which a lot of items is to be either accepted or rejected based on a quality characteristic that can be measured on each item in the lot. A sample of items is drawn from the lot, the quantitative measurement is made on each item sampled, and a decision is made to reject or accept the lot based on these measurements.

Each sampled item is categorised as conforming or nonconforming. The quality of item is determined by some variable X, and an item is considered conforming if $X < u$ (u is given). An attribute plan based the decision to accept or reject the lot only on the number of nonconforming items in the sample. Variables plans use the distribution of X and able to achieve the same control with a smaller sample size. We consider that a distribution of X follows the two-parameter family of distributions depending on unknown shift and scale parameters. To test the hypotheses concerning a proportion of nonconforming items in the lot we consider uniformly most powerful invariant tests, an asymptotic approach and a random size of the sample.

We consider also a problem to compare proportions of nonconforming items in two lots of items.

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The model of hydrodynamic-statistical forecast of the storm wind and of the wind waving over the North and Norway Seas

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The dangerous wind waving over the North and Norway Seas is linear connect with the wind velocity. The velocity $V > 19$m/s have involved the dangerous wave high $h = 3 - 5$m. The prediction of such events is very actual and difficult problem. Nowadays in Russia there is no hydrodynamic model for forecast of the wind with the velocity $V \geq 20$m/s, $V \geq 25$m/s, hence the main tools of objective forecast are statistical methods using the dependence of the phenomena A (the winds with the velocity $V > 19$m/s or $V > 24$m/s) on a number of atmospheric parameters (predictors).

For this purpose the different teaching samples of presence of the event A $\{\mathbf{X}(A)\}$ and presence of the event B (the absence of A) $\{\mathbf{X}(B)\}$ were automatically arranged that include the values of forty physically substantiated potential predictors. Then the empirical statistical method was used the diagonalization of the mean correlation matrix $\mathbf{R}$ of the predictors and the extraction of diagonal blocks of strongly correlated predictors. Thus the most informative predictors for the recognition and for the prediction of these phenomena were selected without loosing information. The statistical decisive rules $F_1(\mathbf{X})$ and $F_2(\mathbf{X})$ for diagnosis and prognosis of the phenomena were calculated for choosing informative vector-predictor. We used the criterion of the Mahalanobis distance and criterion of the minimum of entropy H_{min} by Vapnik-Chervonenkis for the selection of predictors. The most informative and weak depend predictors are:

$(H_{1000}, T_{earth}, V_{700}, Td_{earth}\ U_{850} - U_{925}, Iw, T_{300}, mod(gradT_{earth}),$

Iw – index of the instability of Waiting.

The successful development of the new regional hydrodynamic model (the author Losev V.M.) allowed us to use the prognostic fields of those models for calculations of the discriminant functions $F_1(\mathbf{X})$ and $F_2(\mathbf{X})$ in the nodes of the grid 75x75km and the values of probabilities $P_1(\mathbf{X})$ (depended from $F_1(\mathbf{X})$) and $P_2(\mathbf{X})$ (depended from $F_2(\mathbf{X})$) of dangerous wind $V > 24$m/s (the wind high $h = 5 - 8$m) and thus to get fully automated forecast for the territory of Europe. The author proposes the empirical threshold values specified for these phenomena of the wind with the velocity $V > 19$m/s and of the wind with the velocity $V > 24$m/s and advance period 36 hours over the territory of the Norway and North Seas. According to the Pirsey-Obukhov criterion (T), the success of these automated statistical methods of forecast of storm winds in the warm and cold season for the territory of these Seas is $T = 1 - a - b = 0{,}54 - 78$ after author experiments, where a and b - are the errors of I and II kinds.

A lot of examples of forecasts of storm wind and connected with them wind wavind over the territory of Norway, North and Barents seas are submitted at this report. The rules likes these were applied to the forecast of storm wind over these seas during cold period in this year too (the example of the forecast of the storm wind “St. Iuda” on the 28.10.2013). The great amounts of the velocity and connected with them very high waves of storm wind were observed also at these territories on 1.07.09, on 2–3.08.09, on 18–19.08.09 and other and in cold period 23–27.02.2010, in November 2009. The forecast of these phenomena was given successful with the earliness even 36–48h.

Comparison theorems for small deviations of Green Gaussian processes in weighted L_2-norms

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Suppose we have a zero mean Gaussian process $X(t)$, $0 \le t \le 1$. Let ψ be a non-negative weight function on $[0,1]$. We set

$$\|X\|_\psi = \left(\int_0^1 X^2(t)\psi(t)dt \right)^{1/2}.$$

We establish a comparison of $\mathsf{P}(\|X\|_{\psi_1} \le \varepsilon)$ and $\mathsf{P}(\|X\|_{\psi_2} \le \varepsilon)$ as $\varepsilon \to 0$, when X is a *Green* Gaussian process, i.e. a Gaussian process with covariance being the Green function for a self-adjoint differential operator. This result gives us the opportunity to obtain the sharp small ball asymptotics for many classical processes under quite general assumptions on the weight.

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Multifactor dimensionality reduction method and simulation techniques

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High dimensional data arise naturally in many medical and biological investigations, including genetics. Usually such data are viewed as the value of some random factors $X_1, \ldots, X_n$ and the corresponding response variable Y. For instance, in biological and medical investigations Y describes the health state of a patient. From medical and computational points of view it is very important to find among huge number of factors the collection $X_{k_1}, \ldots, X_{k_r}$ which is responsible for certain complex disease provoking. In our research we concentrate on the new MDR (multifactor dimensionality reduction) method developed in [1]-[3]. To predict Y we use some function f in factors $X_1, \ldots, X_n$. The error functional $Err(f)$ involving a penalty function ψ determines the quality of such f. As the law of Y and $(X_1, \ldots, X_n)$ is unknown we cannot find $Err(f)$. Thus statistical inferences is based on the estimates of error functional. In the mentioned works one can find such statistics constructed by means of a prediction algorithm for response variable and K-fold cross-validation procedure. Besides, the criterion of strong consistency and the central limit theorem (CLT) for the proposed estimates are established.

To illustrate our approach in [4] we discuss the results of simulations to identify the collection of significant factors determining a binary response variable. Different forms of dependence of Y on factors $X_1, \ldots, X_n$ are considered. It is worth to emphasize that in all considered examples for reasonable sample sizes our method permits to identify correctly the collections of significant factors (corresponding to the minimum of prediction error estimates). We demonstrate by graphs the character of stabilization of proposed prediction error estimates' fluctuations as sample size grows. This stabilization of estimates can be explained not only by their strong consistency but also on account of their asymptotic normality. In this regard we formulate the new version of CLT for regularized estimates.

To establish this CLT we prove some limit theorems for row-wise exchangeable random arrays using Lindeberg method and some recent achievements in Stein's techniques in high dimensions [5]. Thus it permits to take statistical estimates of penalty function ψ from a wider class of functions. The statistical variant of our CLT provides the possibility to construct the approximate confidence intervals for unknown errors because we evaluate the variance of the limiting normal law and give the appropriate estimate of this variance.

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Dirichlet heat kernels for rotation invariant Lévy processes

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In this talk I will consider a Lévy rotation invariant process in R^d. Under some weak scaling assumptions about the symbol of the process some estimates of the transition density in terms of the symbol will be presented. Next, sharp estimates of the transition density of the killed process will be described, usually for small values of time. Under global scaling conditions for the symbol, for smooth domains, the obtained estimates are very sharp and they show clear dependence on geometrical characteristics of the underlying domain. In particular they apply to subordinate Brownian motions for which many results of the above type were obtained recently. Even in this case our results are more general then existing ones.

The talk is based on a joint work with Krzysztof Bogdan and Tomasz Grzywny.

Fractional stable statistics in microarray data

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At present time reliable established that probability density functions of gene expression in microarray experiments possess of universal properties. These distributions have power law asymptotic and shape of these distributions are inherent for all organisms [1]. This fact led to appearance of a number works where authors investigate various probability distributions for approximation of empirical distribution of gene expression. In the works have been investigated possibility of usage of such distribution as Poisson, exponential, logarithmic, Zipf-Paretto's distribution and others. In certain works [2] is noted that, for example, double Paretto-lognormal distribution in the best approximation among all listed above distributions of empirical densities.

In this work the fractional stable distributions were used for approximation of gene expression of mcroarray experiments. This distributions are limit distribution of sum independent identical distributed random variable. The probability density function is expressed through Melin's transformation of two stable distribution

$$q(x; \alpha, \beta, \theta) = \int_0^\infty g(xy^{\beta/\alpha}; \alpha, \theta) g(y; \beta, 1) y^{\beta/\alpha} dy,$$

where $g(x; \alpha, \theta)$ and $g(x; \beta, 1)$ are stable and one-sided stable laws respectively. The parameters are varying within limits $0 < \alpha \leqslant 2, 0 < \beta \leqslant 1, -1 \leqslant \theta \leqslant 1$. More detail information reader can find in the work [3]. The main reasons according to which the fractional stable distribution have been used for approximation of gene expressions is that they have power law asymptotic.

As an object of investigation it were chosen gene expression data for following organisms: rat, arabidopsis and Maize leaves, canine, chicken, rice, C. elegans, drosophila, clinical S. aureus strains, P. aeruginosa, Escherichia coli, human , S. cerevisiae. All data were obtained from free database http://www.ebi.ac.uk/arrayexpress/.

The task consists in test of hypothesis about possibility of description of distribution of gene expression by FSD. For this purpose it were chosen gene expression data from CEL files of Affymetrix microarray chips without any prerpocessing. Results obtained from all probes were processed. Obtained data is considered as sample of random variables $Z_1, Z_2, \ldots, Z_N$ and we suppose that each random variable belong to the class of fractional stable laws with parameters $\alpha, \beta, \theta, \lambda$. Since the class are fully defined by their parameters then the task consists in estimation of values $\hat{\alpha}, \hat{\beta}, \hat{\theta}, \hat{\lambda}$ of parameters $\alpha, \beta, \theta, \lambda$ of general population according to the sample $Z_1, Z_2, \ldots, Z_N$. The parameters were estimated according to the algorithm described in [4]. Next, for the parameters have been obtained the probability density function is estimated by histogram method. The results of approximation for human tissues and C. Elegans are presented in the figure 1. As we can see from the figures the fractional stable distribution are good enough approximates of gene expression profiles.

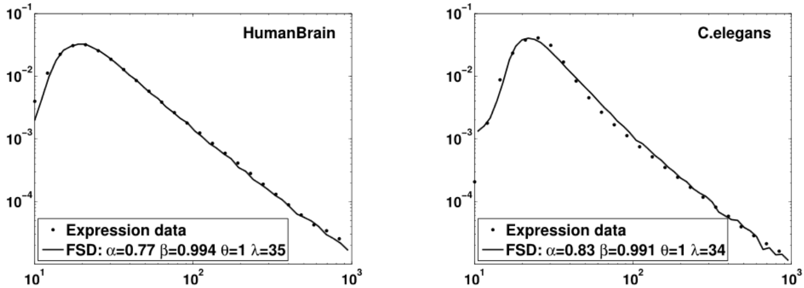

Figure 1: Distribution of gene expression of microarray experiments for human tissues and c.elegans. Solid circles are experimental distribution, solid line is fractional stable distribution. The parameters of distribution are presented on the figure.

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On quasinonuniform estimates for asymptotic expansions in the CLT

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Let $X_1, X_2, \ldots$ are independent identically distributed random variables, $\mathsf{E}X_1 = 0$, $\mathsf{E}X_1^2 = 1$, $\mathsf{E}X_1^6 < \infty$. We denote P the common distribution of these variables, P_n – the distribution of $(X_1 + \ldots + X_n)/\sqrt{n}$, Φ – the standard normal law. We shall consider expansions

$$p_n(x) = \varphi(x) + \varphi(x) \sum_{j=1}^{m-1} \frac{A_j(x)}{n^{j/2}} + O\left(\frac{1}{n^{m/2}}\right), \quad n \to \infty, \tag{1}$$

where $p_n(x)$ are densities of P_n, $\varphi(x) = e^{-x^2/2}/\sqrt{2\pi}$ is the density of Φ, $m = 2, 3, 4$,

$$A_1(x) = \frac{\theta_3}{3!} H_3(x),$$

$$A_2(x) = \frac{\theta_4}{4!} H_4(x) + \frac{n-1}{2n} \left(\frac{\theta_3}{3!}\right)^2 H_6(x),$$

$$A_3(x) = \frac{\theta_5}{5!} H_5(x) + \frac{n-1}{n} \frac{\theta_3}{3!} \frac{\theta_4}{4!} H_7(x) + \frac{(n-1)(n-2)}{6n^2} \left(\frac{\theta_3}{3!}\right)^3 H_9(x),$$

$H_k(x) = (-1)^k \varphi^{(k)}(x)/\varphi(x)$, $k = 0, 1, \ldots$, are Chebyshev - Hermite polynomials, $\theta_k = \int_{-\infty}^{\infty} H_k(x) P(dx)$, $k = 3, 4, 5$, are Chebyshev - Hermite moments of P.

It is clear that from (1) follow the inequalities

$$\left| p_n(x) - \left(\varphi(x) + \varphi(x) \sum_{j=1}^{m-2} \frac{A_j(x)}{n^{j/2}} \right) \right| \le \varphi(x) \frac{|A_{(m-1)}(x)|}{n^{(m-1)/2}} + \frac{R_m}{n^{m/2}}, \tag{2}$$

for $m = 2, 3, 4$. The values R_m in (2) do not depend on n. For all values in (2), except $p_n(x)$, the explicit formulas are known.

It turns out that the first term in the right side of this inequality correctly reflects the behavior of its left part, without the second term in the right side inequality wrong.

The main content of the report is the consideration of the inequality (2) for the cases when the distribution P have the density $e^{-(x+1)}$, $x \ge -1$ (in this case the densities $p_n(x)$ are easily calculated) and $n = 100, 400, 900, 1600$. For this distribution $\theta_3/3! = 1/3$, $\theta_4/4! = 1/4$, $\theta_5/5! = 1/5$ and $R_4 < 2.2 + 70/\sqrt{n} + 220/n$ for $n \ge 100$ and all the values in (2) for $m = 2, 3, 4$ can easily be calculated.

The report is accompanied by numerous graphic illustrations.

Potential theory in hyperbolic space

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In recent years we have seen a considerable growth of interest in hyperbolic Brownian motion. The reason is a strong relationship between this process and some functionals playing an important role in economics (e.g. Asian options), see [1] and [2]. One of the main objects in the theory are the Green function, which measures how much time a process spends in any set, and the Poisson kernel, which describes where a process hits while exiting a fixed set.

We consider the n-dimensional hyperbolic Brownian motion with drift $\{X^{(\mu)}(t)\}_{t\geq 0}$, $\mu > 0$, on the real hyperbolic space $\mathbb{H}^n = \{x \in \mathbb{R}^n : x_n > 0\}$. The generator of the process is the operator $\frac{1}{2}\Delta_\mu$, where

$$\Delta_\mu = \frac{1}{2} x_n^2 \sum_{k=1}^{n} \frac{\partial^2}{\partial x_k^2} - \frac{2\mu - 1}{2} x_n \frac{\partial}{\partial x_n}.$$

Putting $\mu = \frac{n-1}{2}$ we obtain a standard hyperbolic Brownian motion. The subject of our studies is the λ-Green function $G_U^{(\mu),\lambda}(x,y)$ and the λ-Poisson kernel $P_U^{(\mu),\lambda}(x,y)$, $\lambda \geq 0$, of Lipschitz domains $U \subset \mathbb{H}^n$. For U bounded in hyperbolic metric we provide following relationships

$$\text{(1)} \qquad G_U^{(\mu),\lambda}(x,y) = \left(\frac{x_n}{y_n}\right)^{\mu-\eta} G_U^{(\eta),0}(x,y),$$

$$\text{(2)} \qquad P_U^{(\mu),\lambda}(x,y) = \left(\frac{x_n}{y_n}\right)^{\mu-\eta} P_U^{(\eta),0}(x,y),$$

where $\eta = \sqrt{\mu^2 + 2\lambda}$. In fact, the formula for the λ-Green function is valid also for unbounded domains U. In case of λ-Poisson kernel of unbounded sets a different approach is needed. We introduce a new definition of this object and prove modified formula (2).

As an example we give uniform estimates both of the λ-Green function and the λ-Poisson kernel of hyperbolic strip $S_a = \{x \in \mathbb{H}^n : x_1 \in (0,a)\}$, $a > 0$.

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Supremum distribution of Bessel process of drifting Brownian motion

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In his famous paper David Williams [4] showed how to decompose the paths of a transient one-dimensional diffusion at its maximum (or minimum). One of the best-known examples of such decomposition is that of $B(t)+\mu t$, Brownian motion with a positive drift μ, as a Brownian motion with a negative drift $B(t)-\mu t$ and a diffusion Z_t with a generator

$$\Delta_\mu = \frac{1}{2}\frac{d^2}{dx^2} + \mu \coth(\mu x)\frac{d}{dx}. \tag{1}$$

One can construct Z_t in the following way: let $(B_t^{(1)}, B_t^{(2)}, B_t^{(3)} + \mu t)$ be a three-dimensional Brownian motion with drift μ, starting at the origin. Then $Z_t = \|(B_t^{(1)}, B_t^{(2)}, B_t^{(3)} + \mu t)\|$ is a diffusion with generator given by (1).

Process (Z_t) is known as a Bessel process of drifting Brownian motion and denoted $\mathrm{BES}(3,\mu)$ (Pitman and Rogers [2]) or as a hyperbolic Bessel proces (Revuz and Yor [3]). Indeed, if $\mu = 1$ then (Z_t) is a radial part of a hyperbolic Brownian motion in three-dimensional hyperbolic space.

The transition density function of (Z_t) is well-known (cf. Pitman and Rogers [3]) but to our best knowledge, distributions of different functionals of this process have not been examined yet.

We investigate process (Z_t) killed on exiting interval $(0,\, r_0)$ and give a formula describing distribution of $M_t = \sup_{s\leqslant t} Z_s$, the supremum of the process (Z_t). Because the formula is given as an infinite series, we give its exact estimate using elementary functions. Moreover, our method of estimation applied to a function $ss_y(v,t)$ used in a handbook by Borodin and Salminen [1] give very precise estimate of this function.

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A variant of the law of the iterated logarithm for dependent random fields

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Laws of the iterated logarithm are among the most well-known classical limit theorems of probability theory. Starting from theorems by Khintchine, Kolmogorov and Hartman-Wintner, these laws have been generalized many times to dependent sequences, random fields, set-indexed systems of random variables etc. It is known (Wichura [1]) that for multiparameter random systems, such as d-parameter Brownian motion, the upper limit in the law of the iterated logarithm depends substantially on what is the set of indices over which one takes the upper limit. To make this precise, let $\langle t\rangle = t_1 \dots t_d$ for $t \in \mathbb{R}^d$, and $\operatorname{Log}(x) := \log\max\{x, e\}$, for $x > 0$. The notation $t \to \infty$, with $t \in \mathbb{R}^d$, means that $t_1 \to \infty$, ..., $t_d \to \infty$. Then, for a d-parameter Brownian motion $W = \{W_t, t \in \mathbb{R}_+^d\}$, the almost sure upper limit

$$\limsup_{t\to\infty,\ t\in T} \frac{W_t}{\sqrt{2\langle t\rangle \operatorname{Log}\operatorname{Log}\langle t\rangle}}$$

equals 1, if $T = \{(s, \dots, s) : s \geq 0\}$, and equals $\sqrt{d}$ if $T = \mathbb{R}_+^d$. Similar statements hold for partial sums of independent variables having moments of appropriate order. Thus it is natural to ask what is the general form of the law of the iterated logarithm if the set T over which the limit is taken is general enough. This talk provides the answer to this question in the case of stationary associated random fields.

Recall that a random field $X = \{X_j, j \in \mathbb{Z}^d\}$ is called associated if for any $n \in \mathbb{N}$, arbitrary pair of coordinatewise nondecreasing bounded Borel functions $f, g : \mathbb{R}^n \to \mathbb{R}$ and all $i_1, \dots, i_n \in \mathbb{Z}^d$ one has $cov(f(X_{i_1}, \dots, X_{i_n}), g(X_{i_1}, \dots, X_{i_n})) \geq 0$. Associated random systems arise in mathematical statistics, reliability, statistical physics, random measures theory etc. Independent random variables are automatically associated; sufficient conditions for association to hold are known for many other important classes of random systems. There is also a large number of limit theorems describing the belavior of associated random processes and fields, see Bulinski and Shashkin [2] for a detailed account. An important characteristics of a square-integrable stationary associated random field X is its sequence of Cox-Grimmett coefficients

$$u_r(X) = \sum_{j\in\mathbb{Z}^d:|j|\geq r} cov(X_0, X_j),\ r \in \mathbb{N}.$$

Here $|j| = \max_{i=1,\dots,d} |j_i|$, $j \in \mathbb{Z}^d$. The finiteness and appropriate rate of convergence of $\{u_r(X)\}$ to zero, when $r \to \infty$, is a typical condition for a limit theorem to hold (together with moment restrictions).

To state the main result let us introduce some more notation. For a random field $X = \{X_j, j \in \mathbb{Z}^d\}$ and $n \in \mathbb{N}^d$, we set $S_n = \sum_{0<j\le n} X_j$, where the inequalities between multiindices are understood in the coordinatewise sense. Define the norm $\|z\| = \sum_{i=1}^d |z_i|$ in $\mathbb{R}^d$, and for a set $T \subset \mathbb{N}^d$ let $L(T)$ to be the 1-neighborhood of the set

$$\operatorname{Log} T = \{(\operatorname{Log} t_1, \dots, \operatorname{Log} t_d) : t = (t_1, \dots, t_d)\}$$

with respect to that norm. Finally, for $a > 0$ set $R(a) = \cap_{i=1}^d \{t : t_i \ge a\} \subset \mathbb{R}_+^d$.

Theorem. Let $X = \{X_j, j \in \mathbb{Z}^d\}$ be a stationary associated random field. Suppose that $\sup_{j\in\mathbb{Z}^d} \mathsf{E}|X_j|^{2+\delta} < \infty$ for some $\delta > 0$, and that $u_r(X) = O(r^{-\lambda})$ as $r \to \infty$, with some $\lambda > 0$. Then for any $T \subset \mathbb{N}^d$, with probability 1, one has

$$\limsup_{n\to\infty,\, n\in T} \frac{S_n}{\sqrt{2\langle n\rangle \operatorname{LogLog}\langle n\rangle}} = \sigma\sqrt{r},$$

here $\sigma^2 = \sum_{j\in\mathbb{Z}^d} cov(X_0, X_j)$ and

$$r = r(T) = \lim_{a\to\infty} \inf\Big\{\rho > 0 : \int_{L(T)\cap R(a)} \|x\|^{-\rho} dx < \infty\Big\}.$$

In particular, take $b \in (0,1)$ and

$$T = \{n \in \mathbb{Z}^d : n_i \ge \varphi(n_1 \dots n_d),\, i = 1, \dots, d\},$$

where $\varphi(t) = t^{1/d} \exp(-(\log t)^b)$, $t > 0$. Then one has $r(T) = 1+(d-1)b$. Thus such restriction imposed on the indices of partial sums provides the value of upper limit which lies in between the extreme cases, that of a half-line and of a whole positive orthant.

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Some moment estimates for characteristic functions with applications

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Some moment estimates for characteristic functions are derived that are applied to construction of moment-type estimates of the accuracy of the normal approximation to distributions of sums of independent random variables and Poisson random sums. The presented estimates for characteristic functions have an untraditional nonlinear dependence on moments and trigonometric dependence on the argument instead of a polynomial one.

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Solving some open problems on Brownian areas by applying a new extension of Euler's Theorem

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In this talk we restrict ourselves to versions of a standard Brownian motion process $\{B(t), 0 \leq t \leq T\}$ and a standard Brownian bridge process $\{B^0(t), 0 \leq t \leq T\}$ defined on a finite interval $[0, T]$. Consider the random Riemann integrals, which often occur in practice, $A(t) := \int_0^t h(s)B(s)ds$ and $A^0(t) := \int_0^t h(s)B^0(s)ds$, for some continuous deterministic function $h : [0, T] \to \mathbb{R}$. It is shown that for certain choices of $h(s)$, closed-form expressions can be derived for these integrals by applying suitable expansions of Brownian motion and Brownian bridge processes in proper countable coordinate systems (see, e.g., Breiman [1]). This enables one to study the nature of the sample paths of $A(t)$ and $A^0(t)$. More importantly, the exact non-asymptotic probability distributions of $A(t)$ and $A^0(t)$ are derived rigorously. For example, if $h(s) \equiv 1$ we provide a new proof for the known result that $A(t)$ is $N(0, \frac{t^3}{3})$–distributed, and also derive the new result that $A^0(t)$ is $N(0, \frac{t^3}{3}(1 - \frac{3t}{4T}))$–distributed, for all $0 \le t \le T$. In the latter case, Perman and Wellner [2] provides a heuristic proof only for the case $t = T$ and not for $t < T$. Furthermore, if $h(s) = s$, we obtain the new results that $A(t)$ is $N(0, \frac{2t^5}{15})$–distributed and $A^0(t)$ is $N(0, \frac{2t^5}{15}(1 - \frac{5t}{6T}))$–distributed.

From these results interesting conclusions can be made. For example, if $h(s) \equiv 1$, then $\mathrm{Var}(A(T))/\mathrm{Var}(A^0(T)) = 4$ for *all* T, and if $h(s) = s$, then $\mathrm{Var}(A(T))/\mathrm{Var}(A^0(T)) = 6$ for *all* T. Other choices of $h(s)$, which appear in Swanepoel [3], will be considered. In order to calculate the variances mentioned above, we rely on a newly derived extension of a theorem by Euler regarding infinite series of real numbers involving cosines and sines (Swanepoel [4]). The proof of this theorem, which is based on Bernoulli polynomials, will be briefly discussed.

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Estimates of transition densities for jump Lévy processes

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We give upper and lower estimates of densities of convolution semigroups of probability measures under explicit assumptions on the corresponding Lévy measure (non-necessarily symmetric and absolutely continuous with respect to the Lebesgue measure) and the Lévy–Khinchin exponent. We obtain also estimates of derivatives of densities.

Furthermore, for a large class of Lévy measures, including those with jumping kernels exponentially and subexponentially localized at infinity, we find the optimal in time and space upper bound for the corresponding transition kernels at infinity. In case of Lévy measures that are symmetric and absolutely continuous, with densities g such that $g(x) \asymp f(|x|)$ for nonincreasing profile functions f, we also prove the full characterization of the sharp two-sided transition densities bounds of the form

$$p_t(x) \asymp h(t)^{-d} \cdot \mathbf{1}_{|x| \leq \theta h(t)} + t\, g(x) \cdot \mathbf{1}_{|x| \geq \theta h(t)}, \quad t \in (0, t_0), \quad t_0 > 0, \quad x \in \mathbf{R}^d.$$

This is done for small and large x separately. Mainly, our argument is based on new precise upper bounds for convolutions of Lévy measures. Our investigations lead to some interesting and surprising dichotomy of the decay properties at infinity for transition kernels of purely jump Lévy processes.

The joint work with Kamil Kaleta.

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Cure rate quantile regression for censored data with a survival fraction

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Censored quantile regression offers a valuable complement to the traditional Cox proportional hazards model for survival analysis. Survival times tend to be right-skewed, particularly when there exists a substantial fraction of long-term survivors who are either cured or immune to the event of interest. For survival data with a cure possibility, we propose cure rate quantile regression under the common censoring scheme that survival times and censoring times are conditionally independent given the covariates. In a mixture formulation, we apply censored quantile regression to model the survival times of susceptible subjects and logistic regression to model the indicators of whether patients are susceptible.

The mixture cure rate model assumes a decomposition of the failure time as

$$T = \eta T^* + (1 - \eta)\infty,$$

where $T^* < \infty$ denotes the survival time of a susceptible subject, and the indicator η takes a value of 1 if a subject is susceptible, and 0 otherwise. Based on the logistic regression (Farewell, 1982), we can model the susceptibility indicator η,

$$P(\eta = 1|\mathbf{W}) = \pi(\boldsymbol{\gamma}^{\mathrm{T}}\mathbf{W}) = \frac{\exp(\boldsymbol{\gamma}^{\mathrm{T}}\mathbf{W})}{1 + \exp(\boldsymbol{\gamma}^{\mathrm{T}}\mathbf{W})}.$$

For survival times T^*, we take the usual linear regression model

$$\log T^* = \boldsymbol{\beta}^{\mathrm{T}}\mathbf{Z} + \epsilon,$$

where the error ϵ may depend on $\mathbf{Z}$. Given $\tau \in (0,1)$, $Q_{T^*}(\tau|\mathbf{Z}) = \inf\{t\colon P(T^* \leqslant t|\mathbf{Z}) \geqslant \tau\}$ is the τth conditional quantile function, and the quantile regression model is given by

$$Q_{T^*}(\tau|\mathbf{Z}) = \exp\{\mathbf{Z}^{\mathrm{T}}\boldsymbol{\beta}(\tau)\}, \qquad \tau \in (0,1),$$

where $\boldsymbol{\beta}(\tau)$ is an unknown $(p+1)$-vector of regression coefficients.

We develop two estimation methods using martingale-based equations: One approach fully utilizes all regression quantiles by iterating estimation between the cure rate and quantile regression parameters; and the other separates the two via a nonparametric kernel smoothing estimator.

Following the martingale formulation of censored quantile regression in Peng and Huang (2008), we can develop the estimating equation

$$n^{-1}\sum_{i=1}^{n}\mathbf{Z}_i\left\{N_i(\exp\{\mathbf{Z}_i^{\mathrm{T}}\boldsymbol{\beta}(\tau)\}) - \int_0^{\tau} I[X_i \geqslant \exp\{\mathbf{Z}_i^{\mathrm{T}}\boldsymbol{\beta}(u)\}]H_{\boldsymbol{\gamma}}(\mathrm{d}u|\mathbf{W}_i)\right\} = 0,$$

where $H_{\boldsymbol{\gamma}}(u|\mathbf{W}) = -\log\{1 - \pi(\boldsymbol{\gamma}^{\mathrm{T}}\mathbf{W})u\}$ and $N_i(t) = \Delta_i I(X_i \leqslant t)$ for $i = 1, \ldots, n$.

We can extract the cure information to construct an estimating equation for $\boldsymbol{\gamma}$. To avoid the difficulty arising from the entanglement of $\widehat{\boldsymbol{\beta}}(\cdot)$ and $\widehat{\boldsymbol{\gamma}}$, we propose an alternative nonparametric approach based on the locally weighted Kaplan–Meier estimator, which estimates $\boldsymbol{\gamma}_0$ separately from $\boldsymbol{\beta}_0(\cdot)$. Bandwidth selection is often a critical part of nonparametric regression. In practice, we recommend a d-fold cross-validation method for choosing the bandwidth.

We establish the uniform consistency and weak convergence properties for the estimators obtained from both methods. The proposed model is evaluated through extensive simulation studies and illustrated with a bone marrow transplantation data example.

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Application of statistical methods to educational research

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Recently, quite often in the study of various pedagogical phenomena mathematical statistic is used. But using of these methods is not always performed correctly. For example, often to test hypotheses Student's test is used. However, this criterion is applicable only in the case of normal distributions, which in educational research appear rarely. One example of histograms of scores is shown in Fig. 1.

The participants in the current study were undergraduate students enrolled in an introductory course in probability and statistics at Ohio University in Athens, OH. There were 27 total participants. X quizzes were assigned to students to assess their comprehension of course material. In each quiz, one problem was given whose content was discussed during group-work activities, in addition to one problem whose content was taught during a traditional lecture format. We would write the hypothesis H_0: there is no difference between the two comprehension of course material. Let level of significance be 0.05.

By using the t-test statistic in SPSS, we have got the results, which are presented in the Figure 2. The lowest level of significance for Student's t-test is 0.19 and we should accept the hypothesis H_0.

By using the Wilcoxon signed rank test, we have got the results, which are presented in the Figure 3. The lowest level of significance this test is 0.01 and we should reject the hypothesis H_0.

We can do the conclusion, that the result of Student's t-test is wrong, due to the fact, that the distribution of database is not normal.

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
									95% Confidence Interval of the Difference	
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
trad	Equal variances assumed	14,364	,000	-1,318	52	,193	-,48148	,36535	-1,21461	,25165
	Equal variances not assumed			-1,318	42,683	,195	-,48148	,36535	-1,21844	,25548

Figure 1: Student's t-test.

Test Statistics[b]

	gr - tr
Z	-2,586[a]
Asymp. Sig. (2-tailed)	,010

a. Based on negative ranks.

b. Wilcoxon Signed Ranks Test

Figure 2: Wilcoxon signed rank test.

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Uniform truncation bounds for weakly ergodic birth-death processes

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The problem of existence and construction of limiting characteristics for inhomogeneous (in time) birth and death processes is important both for theory and applications. General approach and related bounds for the study on the rate of convergence was considered in [1,2].

Calculation of the limiting characteristics for the process via truncations was firstly mentioned in [3] and was considered in details in [4]. First results for more general Markovian queueing models have been obtained recently in [5].

About two decades ago Vladimir V. Kalashnikov suggested that in some cases one can obtain uniform (in time) error bounds of truncation.

Here this conjecture is studied for a class weakly ergodic birth-death processes.

Let $X = X(t)$, $t \geq 0$ be a birth-death process (BDP) with birth and death rates $\lambda_n(t)$, $\mu_n(t)$ respectively.

Let $p_{ij}(s,t) = Pr\{X(t) = j\,|X(s) = i\}$ for $i, j \geqslant 0$, $0 \leq s \leq t$ be the transition probability functions of the process $X = X(t)$ and $p_i(t) = Pr\{X(t) = i\}$ be the state probabilities.

Secondly we consider the family of "truncated" processes $X_N(t)$ on state space $E_N = \{0, 1, \dots, N\}$, where birth rates are $\lambda_n(t)$, $n \in E_{N-1}$ and death rates $\mu_n(t)$, $n \in E_N$.

By $\mathbf{p}(t) = (p_0(t), p_1(t), \dots)^T$, and by $\mathbf{p_N}(t) = (p_0(t), p_1(t), \dots,_N(t))^T$, $t \geqslant 0$ we denote the column vectors of state probabilities for $X(t)$, and $X_N(t)$ respectively.

We prove the "uniform" approximation bound in the form

$$\|\mathbf{p}(t) - \mathbf{p_N}(t)\|_{TV} \leqslant \frac{C}{g_N}, \quad t \geqslant 0,$$

under assumptions of exponential weak ergodicity of $X(t)$ in special weighted norms, where C is a constant, and $g_N \to 0$ as $t \to \infty$, see details and examples in [6].

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Properties of likelihood ratio test applying for discriminating of Normal and Laplace distributions

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We study the problem of discriminating the Gauss and Laplace distributions by sampling. For the case of composite hypotheses we have shown that the likelihood ratio test is reduced to the Geary test if we substitute the parameters estimates instead of these unknown parameters. We have proved the invariance and have studied the test asymptotic properties for both alternatives.

Moreover, using the software package Wolfram Mathematica 8 for the statistical simulation method, were performed the following calculations for the wide range of changes of the Geary test significance level and for the different volumes of samples:

- test critical points for samples from the normal distribution;
- test power for samples from the Laplace distribution;
- mean and standard deviations estimates for the distribution of the test statistics for samples of the Gauss and Laplace distributions.

Simulation results are written in the relevant tables. Illustrative graphs have been built for the Giri test power, histograms of distributions of Giri statistics (for both alternatives) as well as their approximation of the normal distribution.

Asymptotic distributions of multivariate geometric random sums

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Multivariate geometric random sums were introduced in [1] as the sums of the following type:

$$S = (S_1, \dots, S_k) = \left(\sum_{j=1}^{M_1} X_j^{(1)}, \dots, \sum_{j=1}^{M_k} X_j^{(k)} \right).$$

Let's assume that the random variables included in this expression, imposed the following conditions:

I. The vector $M = (M_1, \dots, M_k)$ has a multivariate geometric distribution (MVG-distribution).

II. For random variables $X_j^{(i)}$ following conditions are satisfied.

   1. $X_j^{(i)}$ are i.i.d. random variables with characteristic functions $\varphi_i(\theta_i) = Ee^{iX_j^{(i)}}$;

   2. $\varphi_i(p^{1/\alpha_i}\theta_i) = 1 + p \ln g_i(\theta_i) + o(p)$ as $p \to 0$,

   where $g_i(\theta_i)$ is the characteristic function of a strictly stable distribution with $\alpha = \alpha_i, \beta = \beta_i, \eta = \eta_i$.

III. M_l and $X_j^{(i)}$ are independent.

Theorem. *Under the conditions of I-III*

$$\overset{o}{S} = (\overset{o}{S}_1, \dots, \overset{o}{S}_k) = (p^{1/\alpha_1} \sum_{(j=1)}^{M_1} X_j^{(1)}, \dots, p^{1/\alpha_k} \sum_{j=1}^{M_k} X_j^{(k)}) \Rightarrow V \ \ as \ \ p \to 0,$$

where V has a general marginally strictly geometric stable distribution (GMS-GSL).

The special parametric family of multivariate distributions, called the general marginally strictly geometric stable law (GMSGSL), which can be uniquely recovered from the univariate distributions of its margins, was introduced in [2] by the following way.

GMSGSL distributions are the distributions of the vector

$$V = (Z_1^{1/\alpha_1} Y_1, \ \dots \ Z_k^{1/\alpha_k} Y_k),$$

where Y_i $(i = 1, \dots, k)$ are independent random variables having strictly stable distributions with characteristic functions $g_i(\theta_i)$ and with the parameters $\alpha_i, \eta_i, \beta_i$; $Z = (Z_1, \dots, Z_k)$ is (independent from $Y_1, ..., Y_k$) random vector having multivariate exponential distribution of Marshall-Olkin (MVE-distribution, see [3]).

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ASSOCIATED WORKSHOP
VIII International Workshop “Applied Problems in Theory of Probabilities and Mathematical Statistics related to modeling of information systems” Summer Session

Stationary distribution of $MMPP|D|1|R$ queue with bi-level hysteric policy

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The rapid development of telecommunication services based on the SIP protocol and growth of number of users have revealed a number of shortcomings in the basic overload control SIP mechanism 503 (Service Unavailable). Being motivated by this problem and by loss-based overload scheme (proposed by IETF SIP Overload Control Working Group for dealing with congestions in SIP network), we consider the generalization of the model introduced in [1]. Specifically, consideration is given to the analysis of queueing system $MMPP|D|1|R$ with bi-level hysteretic input load control. Bi-level hysteretic input load control implies that system may be in three states (normal, overloaded, blocking), depending on the total number of customers present in it, and upon each state change input flow rate is adjusted. The generalization concerns service time (which is considered to be constant instead of exponentially distributed) and number of phases of markov modulated poisson process (which is assumed to be arbitrary integer $1 < n < \infty$).

New method is being proposed (based on approach initially proposed in [2]) for computation of main performance characteristics of the system and calculation of joint stationary distribution at an arbitrary time of number of customers in the queue, elapsed service time and system’s state.

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A discrete-time retrial queueing system with different types of displacements

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In this paper we analyze a discrete-time queueing system in which an arriving customer can decide, with a certain probability, to go directly to the server expelling out of the system the customer that is currently in service or to join the orbit in order to try to reenter at some random time later on. We carry out an extensive analysis of the system.

1. **The Mathematical model**

   Customers arrive according to a geometric arrival process with rate p. If, upon arrival, the service is idle, the service of the arriving customer begins immediately, otherwise, the arriving customer either with probability θ_1 displaces the customer that is currently being served to the head of the orbit and with probability θ_2 expels it out of the system starting immediately, in both cases, its service.

   We will assume that only the customer at the head of the orbit is allowed for access to the server. It is always supposed that retrials and services can be started only at slot boundaries and their durations are integral multiples of a slot duration. Successive inter-retrial times of any customer are governed by an arbitrary distribution $\{a_i\}_{i=0}^{\infty}$ with generating function $A(x) = \sum_{i=0}^{\infty} a_i\, x^i$. Service times are governed by an arbitrary distribution $\{s_i\}_{i=1}^{\infty}$, with generating functions $S(x) = \sum_{i=1}^{\infty} s_i\, x^i$. After service completion, the served customer leaves the system forever and will have no further effect on the system. In order to avoid trivial cases, we assume $0 < p < 1$.

2. **The Markov chain**

   At time m^+ (the instant immediately after time slot m), the system can be described by the process $Y_m = (C_m, \xi_{0,m}, \xi_{1,m}, N_m)$ where C_m represents the server state (0 or 1 according to the server is free or busy, respectively) and N_m is the number of customers in the retrial group.

If $C_m = 0$ and $N_m > 0$, $\xi_{0,m}$ denotes the remaining service time of the customer being served. If $C_m = 1$, $\xi_{1,m}$ corresponds to the remaining service time of the customer being served. Some results of this paper are summarized in the following:

Theorem. *The generating functions of the stationary distribution of the chain are given by*

$$\varphi_0(x,z) = \frac{A(x) - A(\bar{p})}{x - \bar{p}} \frac{p[\bar{p} - s(\bar{p})]\theta_1 xz}{(\bar{p}A(\bar{p}) + \theta_1 z)S(\bar{p}) - \bar{p}\theta_1 z} \pi_{0,0}$$

$$\varphi_1(x,z) = \frac{S(x) - S(\bar{p})}{x - \bar{p}} \frac{p\bar{p}x}{(\bar{p}A(\bar{p}) + \theta_1 z)S(\bar{p}) - \bar{p}\theta_1 z} \pi_{0,0},$$

where

$$\pi_{0,0} = \frac{(\bar{p}A(\bar{p}) + \theta_1)S(\bar{p}) - \bar{p}\theta_1}{A(\bar{p})\big[(\bar{p} + \theta_1)S(\bar{p}) - \bar{p}\theta_1\big] + \bar{p}(1 - S(\bar{p}))}.$$

Corollary. *The probability generating function of the number of customers in the retrial group (i.e. of the variable N) is given by*

$$\psi(z) = \pi_{0,0} + \varphi_0(1,z) + \varphi_1(1,z) =$$

$$= \frac{A(\overline{p})[(\overline{p} + \theta_1 z)S(\overline{p}) - \overline{p}\theta_1 z] + \overline{p}(1 - S(\overline{p}))}{(\overline{p}A(\overline{p}) + \theta_1 z)S(\overline{p}) - \overline{p}\theta_1 z} \pi_{0,0}$$

The probability generating function of the number of customers in the system (i.e. of the variable L) is given by

$$\Phi(z) = \pi_{0,0} + \varphi_0(1,z) + z\varphi_1(1,z) =$$

$$= \frac{A(\overline{p})[(\overline{p} + \theta_1 z)S(\overline{p}) - \overline{p}\theta_1 z] + \overline{p}z(1 - S(\overline{p}))}{(\overline{p}A(\overline{p}) + \theta_1 z)S(\overline{p}) - \overline{p}\theta_1 z} \pi_{0,0}$$

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Stability of retrial queueing system with constant retrial rate

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We study the stability of a single-server retrial queueing system with constant retrial rate and general input and service processes. In such system the external (primary) arrivals follow a renewal input with rate λ. The system also has service times with rate μ. If a new customer finds all servers busy and the buffer full, it joins an infinite-capacity virtual buffer (or *orbit*). An orbital (secondary) customer attempts to rejoin the primary queue after an exponentially distributed time with rate μ_0.

First, we present a review of some relevant recent results related to the stability criteria of similar systems. Sufficient stability conditions were obtained in Avrachenkov and Morozov [1] and have the following form:

$$(\lambda + \mu_0)P_{loss} < \mu_0, \tag{1}$$

where P_{loss} is a stationary loss probability in the majorant loss system. The presented statement holds for a rather general retrial system. However, only in case of Poisson input an explicit expression is provided; otherwise one has to rely on simulation.

On the other hand, the stability criteria derived in Lillo [2]

$$\frac{\lambda(\mu+\mu_0)^2}{\mu\Big[\lambda\mu[1-C(\mu+\mu_0)]+\mu_0(\mu+\mu_0)\Big]} < 1, \tag{2}$$

where

$$C(s) \quad = \quad \int_0^\infty e^{-xs}dF(x),\ s>0 \tag{3}$$

can be easily computed, but hold only for the case of exponential service times.

We present new sufficient stability conditions, which are less tight than the ones obtained in Avrachenkov and Morozov [1], but have an analytical expression under rather general assumptions. A key assumption is that the input intervals belong to the class of *new better than used* (NBU) distributions. The new condition is based on the connection between P_{loss} and P_{busy} (stationary busy probability) in the majorant loss system. This statement was obtained in Morozov and Nekrasova [3] and can be expressed as:

$$P_{loss} = 1 - \frac{1}{\rho} P_{busy}. \quad (4)$$

We also illustrate the accuracy of these conditions (in comparison with known conditions when possible) for a number of non-exponential distributions.

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Queuing model of resource allocation in LTE uplink channel

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LTE has been devised by the 3GPP to improve end-user throughput, reduce user plane latency and cope with the increasing demand for better Quality of Service (QoS) [1]. In uplink LTE exploits Single Carrier-Frequency Division Multiple Access scheme, which requires that all the Resource Blocks (RBs) assigned to the same User Equipment (UE) must be contiguous in frequency domain. Note that LTE specification does not recommend any uplink Resource Allocation Algorithm (RAA) by the evolved Node B (eNB). However, there are a number of mechanisms defined in the LTE network that allow performing uplink scheduling efficiently, e.g. Sounding Reference Signal (SRS) that carries the Channel State Information (CSI) for each RB for each UE, Buffer Status Report (BSR), knowledge of the QoS requirement for each of the session. Uplink RAAs may be classified by principles of channel dependency and proportional fairness (PF). Among the channel-unaware algorithms, Fair Work Conserving [2] outperforms others by its strategy to schedule all the RBs in every subframe (1 ms). The channel-aware schemes [3] achieve the best throughput, but suffer from starvation problem that can be solved by means of proportional fairness paradigm [4]. However, both channel-aware and PF schemes do not take the UE QoS requirements into consideration, which may result in scenario when the UE with highest priority and lowest channel quality may not get enough RBs to fullfill the QoS requirement. In this paper we introduce an analytical model that takes into account SRS, BSR and QoS requirements, and allows analyzing various uplink RAA by means of performance measures evaluation.

Description of the model. The structure of the proposed analytical model is shown in Fig. 1. Let us assume that there are M UEs in LTE cell that may initialize the session of uplink transmission, whereas eNB has N RBs available for distribution. The system functions in discrete time with the constant slot length h of 1 ms, and all the changes in the system occur at moments $nh, n = 1, 2, ...$ We consider two types of sessions: with priority (0), and without priority (1). Being empty, the UE$_i$ (here and further, $i = \overline{1, M}$) can initialize a new session during a time slot with the probability a_i. By opening a session a number of bits prepared for transmission is generated at the UE$_i$ buffer of r_i capacity. Note that a new session belongs to a prioritized 0-type with probability d_i, and to the non-prioritized 1-type with $\overline{d_i} = 1 - d_i$. The parameter $c_i = \{0, 1\}$ models the priority type of the UE$_i$ in the current time slot. UE$_i$ keeps its priority till all bits of the current session will be send from the buffer. In order to analyze channel-aware RAA, we consider that the CSI s_i of UE$_i$ is known for every RB every time slot. Let the buffer occupancy of UE$_i$ in current slot be q_i. During each slot a certain number of packet units of the UE$_i$ may be serviced according to group deterministic distribution D^G, which will lead to reduction of the buffer occupancy. Note that the number of serviced packet units directly depends on the selected RAA, and in general case depends on the CSI, buffer occupancy and type of the session. The described system can be denoted as $Geom^G(\vec{q}) \mid D^G(RAA) \mid N \mid \vec{r} \mid \vec{f_1}$. The functioning of the system is described by the homogeneous Markov chain ξ_n at time moments $nh + 0, n \leq 0$, with the state space: $X = \{(\vec{c}, \vec{q}, \vec{s}) : \vec{c} = (c_1, c_2, ..., c_M)^T, \vec{q} = (q_1, q_2, ..., q_M)^T, \vec{s} = (s_1, s_2, ..., s_M)^T, c_i = \overline{0, 1}, q_i = \overline{1, r_i}, s_i = \overline{1, S^N}\}$, where S - is the overall number of CSI possible values and $s_i \in \boldsymbol{S} := \{(s_i(RB_1), ..., s_i(RB_N)) : s_i(RB_j) \in \{1, 2, ..., S\}, j = \overline{1, N}\}$, $\mid \boldsymbol{S} \mid= S^N$. We assume that the set $\boldsymbol{S}$ is lexicographically ordered. Our contribution in this paper is to use the described system as a framework for analyzing various RAA, e.g. [2-4], by means of performance measures evaluation.

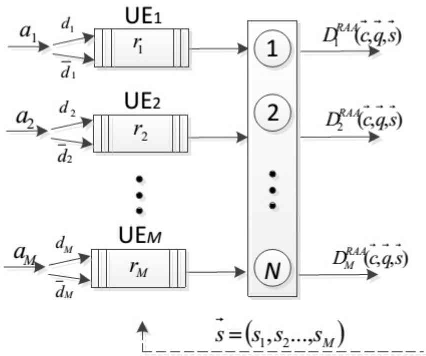

Figure 1: Structure of the model.

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Mathematical models of binary file compression optimality

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Different lossless and lossy compression techniques are very important in modern informatics. In both cases, the compression is an encoding of original files to be compressed. Typically the compression is based on a mapping of original file F into set (c_i, i) of bits which can be, for example, coefficients of spectral transformations (Discrete Fourier, Walsh-Hadamard (WHT)) with their indexes or pairs of indexes for a dictionary based compression techniques [1]. For example, in LZ77 compression, the *longest phrases* with their indexes can be considered as the set (c_i, i) mentioned above. The system designers are interested in the optimality of data compression in order to achieve the minimum number of bits for storing the input string, without assumptions on the generating source statistics. The compression procedure should be efficient in terms of the required time and space. To evaluate the efficiency of compression there is a need for corresponding mathematical models that can assist in information systems efficiency analysis. However, currently such models are very generalized from the point of view of different specific tasks [2], for example, for on-line data compression. In this presentation we consider models for the output files length optimization.

In dictionary-based data compression techniques, any strings of symbols are represented by an index to a dictionary constructed from the source alphabet. The dictionary coding is based on maintaining a dictionary that contains frequently occurring phrases (substring of symbols), in contrast to Huffman coding which is based on computing the symbols occurrence probabilities. When these phrases are encountered and found in the dictionary, they are encoded with an index in the dictionary.

It is well-known [2] that parsing of n-words file can be computed in $O(n)$ time and $O(n_w)$ space, where n_w is the the dictionary size. In this case the greedy parsing is optimal with respect to the number of phrases (corresponding to the indexes (c_i, i)) in which string S can be parsed by the dictionary (called also as the sliding window of size n_w). An important theoretical result in this scope is that such dictionary-based parsing achieves asymptotically the best compression possible and therefore acts (asymptotically) according to the empirical entropy. However, the optimality in the number of parsed phrases is not necessarily equal to the optimality in the number of bits of a compression of a given string S [2].

As we deal with finite strings (files) while Lempel-Ziv theory has been formulated in asymptotic terms, we should consider empirical entropy [3], estimated over corresponding finite data set. In general, following [3], it can be easily shown, that the length L_c of the compressed string can be estimated as $L_c \geq L_{orig} H_{k,m}$, where L_{orig} is the length (number of bits) of the original file, $H_{k,m}$ is empirical entropy estimated using a sliding window of size m, when k is the number of longest matches for given string (block, file). The empirical entropy can be expressed in terms of longest matches and dictionary size as $H_{m,n} = ((1/k) \sum_{i=1}^{k} L_{i,m} / \log_2(m))^{-1}$, that allows to define the problem of the compressing process optimization.

As for Walsh-Hadamard Transformation, this approach to data compression is a lossy one, the number of the compressed bits depends on the quality of possible decompression. In [4] it has been shown that the Hamming distance between original and reconstructed binary files as a *blurriness* measure [4]. We suggested a metric [4] that captures the difference of the bits b_i of the original file and bits b_i, where $i = 1, \ldots, n$, n is the number bits, reconstructed from a truncated set of WHT coefficients. Each coefficient c_i is transmitted/stored with its index i in the WHT matrix, namely the pairs (c_i, i) are stored as the representation of the data. Note, that this truncation enables a compression of the original file (along with other useful features of the WHT [4]). A compression optimization model for reconstruction a binary sequence from a truncated WH series can be also based on an *entropian* paradigm. In particular, we may consider the theoretic Shannon bound $R = -D \log_2(D) - (1 - D) \log_2(1 - D)$, where D is the fraction (probability) of the correctly reconstructed bits and R is the number of bits per symbol transmitted. Note that lossless algorithms provide the compression of a file to the values dependent asymptotically on entropy of a source modeling the file [2], whereas relationship between the entropy and the holographic property of the WHT based codes is not asymptotic.

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Analytical modeling of P2P streaming network

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Here, we propose an approach for modelling the data exchange process between users in P2P streaming network with buffering mechanism in form of discrete Markov chain. Our model takes into account all important characteristics of a P2P streaming network and allows us to evaluate the main performance measures, such as playback continuity and startup delay.

The data exchange mechanism in streaming P2P networks is similar to that of file sharing P2P networks based on the most popular protocol BitTorrent. However, in streaming P2P networks the time window when a peer still needs a data chunk is of critical importance, as every chunk has its playback deadline.

A user downloads data chunks from both the server and other users, who have already downloaded them. For this reason, users constantly exchange buffer maps, providing the information about data availability to one another. This way a user can download one or more missing data chunks from other users.

Note, that a user, who just connected to the network, does not provide any data chunks to exchange with other users and compete with other users for downloading the available data chunks in the network. When a user disconnects from the network, he deprives other users of the opportunity to download anything from him. Thus, the overall performance of the network degrades due to peer churn. We take this into account by introducing the probabilities α and β of a user connecting and disconnecting from the network.

The data chunk exchange process is also affected by so-called lags the data transmission delay between the server and users. Lags define the playback time difference between any two users. Due to playback lags, one and the same data chunk in the buffers of users will be located in positions with different indexes, leading to narrowing the number of data chunks available to exchange between these two users.

The maximum download and upload rate of a user affect the performance of the network as well. Every user will try to use all his download capability in the most effective way by downloading different data chunks from different users; however, the upload speed limitation will make it impossible to download every single data chunk in the network during one time slot. In order to choose which data chunk to download next a download strategy, such as Rarest First (RF), Latest First (LF) or Greedy (Gr), is applied. Download strategies can greatly improve one or another performance measure, for example, RF strategy strives to enhance the overall performance of the network, while Gr strategy reduces the playback startup delay.

Therefore, we propose our model in the next form:

$$\mathbf{Z} = \langle \mathrm{N}, \mathrm{M}, \boldsymbol{\alpha}, \boldsymbol{\beta}, \mathbf{lag}, \mathbf{d}, \mathbf{u}, \delta \rangle .$$

Here [1-4]:

- N is the maximum number of users in the netowrk;
- M is the buffer size of each user;
- $\boldsymbol{\alpha} = (\alpha(1), ..., \alpha(\mathrm{N}))$ is a vector, describing the probabilities with which a new user can join the netwrok;
- $\boldsymbol{\beta} = (\beta(1), ..., \beta(\mathrm{N}))$ is a vector, describing the probabilities with which a user can leave the netwrok;
- $\mathbf{lag} = (lag(1), ..., lag(\mathrm{N}))$ is a vector, that describes the data transmission delays between the server and each user;
- $\mathbf{d} = (d(1), ..., d(\mathrm{N}))$ and $\mathbf{u} = (u(1), ..., u(\mathrm{N}))$ vectors contain the download and upload rate for each user.

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On the techniques for improving efficiency of programming modules for stochastic modelling and simulation

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In the paper we discuss possible ways to improve a performance of realizations of various stochastic models. The main purpose of our work is finding effectiveness techniques for the problem of finite mixture decomposition in compound Cox model, etc. (see, for example, the book [1]).

Undoubtedly, it can be embodied by special programming solutions, e.g., we can realize computational modules using any low-level programming language. The source code might be very effective and fast but too difficult for programming and especially for debugging. Moreover, it needs much time and in fact you have to create a new information technology. We consider approaches of possible optimizations of the existent solutions (see, for example, the paper [2]).

The first way is simply to use more actual and efficient hardware. For example, we have used the newest CPU and obtained up to 3 times acceleration of working with one spectrum comparing our previous hardware. At the same time, this CPU is available for the most of users even in their homes. The modern CPUs have more than one logical core and so you can process multiple data sets simultaneously. The ratio between velocity and time is not linear but you obtain significant acceleration even without special programming solutions!

The second way logically follows the first one in terms of parallelism. Modern integrated development environments support mechanisms for automatization of parallel computing for a source code. Using special directives, program can work faster without wide modifications of the code.

The third way is based on new hardware ideas and creating special source code for these purposes. It leads to computing on GPUs, clusters, etc. At that, GPU solutions are not so expensive as clusters and supercomputers. The world leading GPU producers offer special solutions for researchers in different ares (CUDA technology by NVIDIA, ATI Stream Technology by AMD). It should be noted that in modern GPUs the number of cores equals from several hundred to thousands ones. Obviously, their performance may be extremely high for various complex computational problems in the areas with the critical requirements for accuracy and processing time. Surely, one of the most important problems is a creation of an effective software that would be able to use the maximum power of the hardware solutions. In fact the optimal application performance on multi-core systems can be achieved through rational use of program threads for the correct allocation of subproblems. Threads execution can be optimized for running on a different physical cores.

One of the most important issues is a software's effectiveness to use the sizeable part of hardware performance. Indeed, the optimal application performance on multi-core systems can be achieved only through rational use of program threads for the smart allocation of software problems. Threads can be optimally run on various physical cores to improve system's performance.

Developing this type of software we should follow the principle of decomposition [3], i.e., we try to allocate parts of main problem that can be executed in parallel. There are some types of the decomposition: by problems, by data and by the information flows.

In the first type of decomposition we should use different threads for various tasks. It is the easiest way to create parallel programs involving the simultaneous execution of problems which can be considered as independent with each other. For example, the program can estimate the model parameters, but at the same time user can work with initial data, graphs, etc.

In the second variant of the decomposition different processes handle unique data blocks. For example, you can divide original sample among few various processes based on different models.

The most difficult type in terms of parallelism is decomposition by information flows. In real problems the output of one subtask is often input to another one. Obviously, the second process can not be executed without data from the first one.

Choosing a specific type of decomposition or some hybrid versions, we can realize effective programming solutions. At the same time to solve specific problems you may need an individual approach because in parallel programming technologies we have not only opportunities to increase performance but also some specific requirements for software developers.

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Joint stationary distribution in infinite capacity $MAP|PH|2$ queue with resequencing

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Simultaneous processing systems where the order of customers (jobs, units) upon arrival has to be preserved upon departure may suffer from the impacts of resequencing, thay may be performed inside the systems. Various analytical methods and models have been proposed to study the impacts of resequencing. General survey of queueing theoretic methods and early models for the modeling and analysis of parallel and distributed systems with resequencing can be found in [1], whilst survey on the resequencing problem that covers period up to 1997 can be found in [2]. Among recent related papers related to this topic one can cite [3-7].

In this paper we study the generalized version of the problem considered in [8]. Specifically we consider queueing system with two servers, infinite capacity buffer (for storing customers before they get served) and resequencing buffer (RB) of infinite capacity. New customers arrive at the system according to Markovian arrival process, upon entering the system obtain sequential number and join buffer. Customers leave the system strictly in order of their arrival (i.e. in the sequence order). Thus after customer's arrival it remains in the buffer for some time and then receives service when one of the servers becomes idle. If at the moment of its service completion there are no customers in the system or all other customers present at that moment in the queue and the rest two servers have greater sequential numbers it leaves the system. Otherwise it occupies one place in the RB. Customer from RB leaves it if and only if its sequential number is less than sequential numbers of all other customers present in system. Thus customers may leave RB in groups. Service times of customers on both servers follow the same phase-type distribution.

Efficient method is proposed for computation of joint stationary distribution of the number of customers in buffer and RB. In order to check theoretical results there was built a simulation model. The comparisons of numerical and simulation results showed good accuracy.

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Modelling of SIP server with hysteretic overload control and K-state MMPP input flow

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Major standards organizations, ITU, ETSI, and 3GPP have all adopted SIP as a basic signalling protocol for NGN. The current SIP overload control mechanism is unable to prevent congestion collapse and may spread the overload condition throughout the network [1–3, 6]. IETF work group is developing loss based overload control scheme which should substitute the existing mechanism [1]. The most common implementation of the schema involves the threshold-based load management as an essential tool in prevention of various types of congestions in SIP networks [1, 2]. A variation of the threshold management is a hysteretic mechanism, which uses two types of thresholds to control congestion – congestion onset threshold and congestion abatement threshold. Criteria for the determination of SIP server congestion status are the number of messages in the queue for CPU service, i.e. buffer occupancy. In papers [2–6] some queuing models with Poisson input flow and hysteretic mechanism were introduced and their performance measures were analysed. However, these models do not allow to investigate the performance indicators of a SIP server in the case of bursty input message flow. In this paper we constructed an analytical model of SIP server with $MMPP_K, K \geqslant 2$, input flow and bi-level hysteretic overload control mechanism.

Model Definition. Let us assume that customers arrive at a single-server queue and receive service in accordance with FCFS policy. The processing times are exponentially distributed with the mean μ^{-1}. The server operates in three modes: normal ($h = 0$), overload ($h = 1$), and discard ($h = 2$), where h is the overload status. When the queue length n increases and exceeds the threshold, H, in the normal mode, the system detects the overload and switches to the overload mode. In the overload mode, the system informs the sender about reduction of input load by the probability q. Thereafter, if the queue length decreases and drops below the threshold, L, in the overload mode, the system detects the elimination of overload, turns to normal mode and inform the sender about lifting of the restrictions. If in the overload mode the queue length continues increasing and reaches threshold, R, the system turns to the discard mode and informs the sender about suspension of message dispatching. After that, the queue length starts decreasing in the discard mode and when it drops below the threshold, H, the system detects mitigation of overloading, turns to the overload mode. A Markov modulated Poisson arrival process (MMPP) with $K \geqslant 2$ phases is completely determined by infinitesimal operator $\mathbf{Q} = (q_{ij})_{i,j=\overline{0,K-1}}$ and rate matrix $\mathbf{\Lambda} = diag\,(\lambda_0, \ldots, \lambda_{K-1})$. The changes of input load in the dependency of the systems' states is specified by the following relation

$$\lambda_k^h(n) = \begin{cases} \lambda_k^0, h = 0, & 0 \leqslant n \leqslant H, \\ \lambda_k^1 = (1-q)\,\lambda_k^0, & h = 1, L \leqslant n \leqslant R-1, \\ 0, & h = 2, H+1 \leqslant n \leqslant R. \end{cases}$$

The described system can be denoted as $MMPP_K|M|\langle L, H\rangle|R$. The functioning of the system is described by the Markov process $\mathbf{X}(t) = (h(t), n(t), k(t))$ over the state space $\mathcal{X} = \mathcal{X}_0 \cup \mathcal{X}_1 \cup \mathcal{X}_2$, $\mathcal{X}_0 = \{(h, n, k) : h = 0, 0 \leqslant n \leqslant H-1, 0 \leqslant k \leqslant K-1\}$,

$\mathcal{X}_1 = \{(h,n,k) : h = 1, L \leqslant n \leqslant R-1, 0 \leqslant k \leqslant K-1\}$, $\mathcal{X}_2 = \{(h,n,k) : h = 2, H+1 \leqslant n \leqslant R,\ 0 \leqslant k \leqslant K-1\}$. We introduce the lexical order for set $\mathcal{X}$ and construct infinitesimal operator of process $\mathbf{X}$ in a block-diagonal form. The algorithm from [5] is applied for performance evaluation.

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A cloud computing system with batch arrivals model

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In cloud computing system a user sends a query, which is handled by virtual cloud servers [1-2]. In the system studied here it is assumed that when entering the system the customer query is split into several independent sub-queries according to the number of the cloud computing service providers and each provider handles exactly one sub-query [3]. All sub-queries of the same query are handled simultaneously by the service providers. Notice that unlike [4] the number of sub-queries doesn't have to be exactly equal to the number of service providers. It reflects the situation when a customer query doesn't need some kind of service or when the necessary service is received from another provider. We study response time as a performance metric of the system. Response time is denoted as the maximal within a single query sub-query handling time. We develop a queuing system model with multiple queues and batch arrival to analyze the response time of the cloud computing system.

In this model we assume the external arrival of customers to be Poisson distributed with a rate of λ and the service times at the providers virtual servers are exponentially distributed with a rate of $\mu_k, k = 1, ..., K$. Here K is a number of cloud computing service providers. Furthermore a probability vector $\boldsymbol{l}= (l_1, ..., l_K)$ is introduced, l_k element of this vector determines the probability that the k-th provider is involved in handling the query. A number of virtual servers in the system of the k-th provider is v_k, a queue length of the k-th provider is r_k, $k = 1, ..., K$. The queuing system reflecting the described cloud computing system is presented in Figure 1.

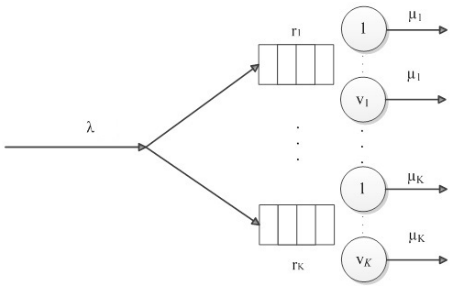

Figure 1: Queuing model of the cloud computing system.

For the described system a transfer rate infinitesimal matrix was obtained. Using a determined lexicographic order

$$\mathbf{n}' > \mathbf{n}'' \Leftrightarrow \left((n'_{\cdot} > n''_{\cdot}) \cup \left((n'_{\cdot} = n''_{\cdot}) \cap \left(\sum_{i=1}^{K} (n'_k > n''_k)(\gamma + 1)^{K-k} > 0 \right) \right) \right)$$

it was proved that the obtained matrix have a block-diagonal form and the formulas to calculate its blocks were also obtained. Here $\mathbf{n}'$ and $\mathbf{n}''$ are vectors describing the number of the sub-queries in the system of each provider. Each element of these vectors n'_k (n''_k) describes the number of sub-queries in the system of k-th provider, $n'. = \sum_{k=1}^{K} n'_k$ ($n''. = \sum_{k=1}^{K} n''_k$) is a sum of all the sub-queries in the system, $\gamma = \max_k(r_k + v_k)$ is the maximal number of sub-queries that can be presented in the largest provider system. Finally an equilibrium equation system was deduced and solved and the average response time of the system was calculated for different initial data values. The results show average response time of each provider as well as the responce time of the whole system.

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Calculating Performance Measures of Pre-Emption Model for Video Conferencing in LTE Network

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LTE networks deployment is inseparably linked with enhancing the quality of service (QoS). LTE operators have to develop and select an optimal radio admission control (RAC) scheme [1,2] accounting for the service level agreement, as the 3GPP recommendations (TS 36.300, TS 23.401, TS 23.203) do not specify such schemes. RAC could be realized through the service degradation referred as partial pre-emption or full pre-emption, i.e. service interruption of lower priority services. In the paper, we propose a full pre-emption based RAC scheme for video conferencing.

We consider a single cell with a total capacity of C bandwidth units (b.u.) supporting two guaranteed bit rate services: multicast multi-rate video conferencing (VC) service (higher priority) and unicast video on demand (VoD) service (lower priority). The VoD service is provided on single guaranteed bit rate $d = 1$ b.u. The VC bit rate can adaptively change from a maximum value of b_1 b.u. to a minimum value of b_K b.u. according to a given set of values $b_1 > \ldots > b_K$, which depends on the cell load expressed in the number of users. Let incoming flows be Poisson of rates λ (VC) and ν (VoD), and the service times be exponentially distributed with means μ^{-1} (VC) and κ^{-1} (VoD). Then we denote the corresponding offered loads as $\rho = \lambda/\mu$ and $a = \nu/\kappa$.

The VC priority level is higher then the VoD one. First, this fact is realized by the adaptive change of the VC bit rate. Second, the admission control is achieved such that a new VC request is accepted by the so-called pre-emption owing to the lack of free cell resources. Pre-empting refers to the release of cell resources occupied by VoD service. Let n be the number of VoD users and let m the state of a multicast session, where m can be equal to 1 if the session is active, i.e. multicast VC service is provided at least to one user on bit rate b_k, $k = 1, \ldots, K$ or m can be equal to 0 if the session is not active. Then the system state space is defined as $\mathcal{X} = \{(0,n):\ n = 0, \ldots, C\ \vee (1,n):\ n = 0, \ldots, C - b_K\}$

The main performance measures of the pre-emption based RAC model are blocking probability B, pre-emption probability Π, and mean bit rate $\bar{b}$:

$$B = [p(0,C) + p(1, C - b_K)] \cdot G,$$

$$\Pi = \left[\sum_{n=C-b_K+1}^{C-1} \frac{\lambda}{\lambda+\nu+n\kappa} \frac{b_K - C + n}{n} p(0,n) + \frac{\lambda}{\lambda + C\kappa} \frac{b_K}{C} p(0,C)\right] \cdot G,$$

$$\bar{b} = \left[b_1 \sum_{n=0}^{C-b_1} p(1,n) + \sum_{k=2}^{K} b_k \sum_{n=C-b_{k-1}+1}^{C-b_k} p(1,n)\right] \cdot G,$$

where

$$G = \left[\sum_{n=0}^{C} p(0,n) + \sum_{n=0}^{C-b_K} p(1,n)\right]^{-1},$$

the unnormalized probability $p(m,n)$ that the system is in state (m,n) can be computed as follows

$$p(m,n) = \alpha_{mn} + \beta_{mn} \cdot x, \quad (m,n) \in \mathcal{X},$$

where

$$x = \frac{\frac{\nu}{\lambda+C\kappa}\alpha_{0,C-1} - \alpha_{0C}}{\beta_{0C} - \frac{\nu}{\lambda+C\kappa}\beta_{0,C-1}},$$

and coefficients α_{mn} and β_{mn} are calculated by recursive formulae

$$\alpha_{00} = 1, \quad \beta_{00} = 0, \quad \alpha_{10} = 0, \quad \beta_{10} = 1,$$

$$\alpha_{01} = \frac{\upsilon + \lambda}{\kappa}, \quad \beta_{01} = -\frac{\mu}{\kappa}, \quad \alpha_{11} = -\frac{\lambda}{\kappa}, \quad \beta_{11} = \frac{\upsilon + \mu}{\kappa},$$

$$n\alpha_{0n} = (\alpha_{01} + (n-1))\,\alpha_{0,n-1} + \beta_{01}\alpha_{1,n-1} - a\alpha_{0,n-2}, \; n = 2, \dots, C - b_K + 1,$$

$$n\beta_{0n} = (\alpha_{01} + (n-1))\,\beta_{0,n-1} + \beta_{01}\beta_{1,n-1} - a\beta_{0,n-2}, \; n = 2, \dots, C - b_K + 1,$$

$$n\alpha_{1n} = (\beta_{11} + (n-1))\,\alpha_{1,n-1} + \alpha_{11}\alpha_{0,n-1} - a\alpha_{1,n-2}, \; n = 2, \dots, C - b_K,$$

$$n\beta_{1n} = (\beta_{11} + (n-1))\,\beta_{1,n-1} + \alpha_{11}\beta_{0,n-1} - a\beta_{1,n-2}, \; n = 2, \dots, C - b_K,$$

$$n\alpha_{0n} = (\alpha_{01} + (n-1))\,\alpha_{0,n-1} - a\alpha_{0,n-2}, \; n = C - b_K + 2, \dots, C,$$

$$n\beta_{0n} = (\alpha_{01} + (n-1))\,\beta_{0,n-1} - a\beta_{0,n-2}, \; n = C - b_K + 2, \dots, C.$$

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Calculating mean service downtime for a model of eNodeB failure in LTE network

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4G LTE wireless networks give the possibility for mobile operators to offer a wide range of multimedia services. The 3GPP specifications define quality of service requirements for LTE networks and typical scenarios that are recommended for network planning and design. Unfortunately, these scenarios don't take into account some factors putting the LTE base station (eNodeB) out of normal mode. This results in temporal unavailability of physical resource blocks. Thus, the process of modeling and analyzing LTE networks should include the possible failures of eNodeBs [1]. We propose a model of eNodeB failures with the finite buffer as a multi-service queuing system with unreliable servers.

Our model is based on the model of a single cell with a total capacity of C bandwidth units supporting a guaranteed bit rate service, i.e. telephony

[2]. The eNodeB fails with rate α. After the failure all users will perceive the temporal service interruption. The information about the current state of serving users will be stored in buffer of a finite size r. The eNodeB is repaired with rate β and the interrupted users will receive the service. We assume failures and repairs of eNodeB be exponentially distributed, the incoming flow be Poisson of rate λ, and the service durations be exponentially distributed with mean $1/\mu$.

Let n be the number of users waiting to receive the service and let m be the number of users receiving the service. It could be obtained that the process representing the system states is not a reversible Markov process and solution $p\,(n,m)$ of the equilibrium equations is not of product form. So, we propose the recursive algorithm for calculating the mean service downtime, i.e. mean waiting time and mean delay.

The mean service downtime can be computed as

$$W = \frac{1}{\lambda\,(1 - q\,(r-C, C) - q\,(r,0))}\left(\sum_{n=1}^{r-C} nq\,(n,C) + \sum_{n=1}^{r} nq\,(n,0)\right),$$

where unnormalized probability distribution $q\,(\cdot,\cdot)$ is calculated as

$$q\,(n,m) = A_{nm}\frac{\lambda\alpha}{\mu(\lambda+\beta)} + B_{nm},$$

$$(n,m) \in \left\{(n,m) : (0,m)\,,\ m=\overline{0,C};\ (n,C)\,,\ n=\overline{1,r-C};\ (n,0)\,,\ n=\overline{1,r}\right\},$$

and coefficients A_{nm}, B_{nm} satisfy following recursion:

$$A_{00} = 0, \quad B_{00} = 1, \quad A_{01} = 0, \quad B_{01} = \frac{\lambda}{\mu}, \quad A_{10} = 1, \quad B_{10} = 0,$$

$$A_{0n} = \left(\frac{\lambda+\alpha}{n\mu} + \frac{n-1}{n}\right)A_{0,n-1} - \frac{\lambda}{n\mu}A_{0,n-2} - \frac{\beta}{n\mu}A_{n-1,0}, \quad n=\overline{2,C},$$

$$B_{0n} = \left(\frac{\lambda+\alpha}{n\mu} + \frac{n-1}{n}\right)B_{0,n-1} - \frac{\lambda}{n\mu}B_{0,n-2} - \frac{\beta}{n\mu}B_{n-1,0}, \quad n=\overline{2,C},$$

$$A_{n0} = \frac{\alpha}{\beta+\lambda}A_{0n} + \frac{\lambda}{\beta+\lambda}A_{n-1,0},\ B_{n0} = \frac{\alpha}{\beta+\lambda}B_{0n} + \frac{\lambda}{\beta+\lambda}B_{n-1,0}, \quad n=\overline{2,C},$$

$$A_{1C} = \left(\frac{\lambda+\alpha}{C\mu} + 1\right)A_{0C} - \frac{\lambda}{C\mu}A_{0,C-1} - \frac{\beta}{C\mu}A_{C0},$$

$$B_{1C} = \left(\frac{\lambda+\alpha}{C\mu} + 1\right)B_{0C} - \frac{\lambda}{C\mu}B_{0,C-1} - \frac{\beta}{C\mu}B_{C0},$$

$$A_{1+C,0} = \frac{\alpha}{\beta+\lambda}A_{1C} + \frac{\lambda}{\beta+\lambda}A_{C0}, \quad B_{1+C,0} = \frac{\alpha}{\beta+\lambda}B_{1C} + \frac{\lambda}{\beta+\lambda}B_{C0},$$

$$A_{nC} = \left(\frac{\lambda+\alpha}{C\mu}+1\right) A_{n-1,C} - \frac{\lambda}{C\mu} A_{n-2,C} - \frac{\beta}{C\mu} A_{n-1+C,0}, \quad n = \overline{2, r-C-1},$$

$$B_{nC} = \left(\frac{\lambda+\alpha}{C\mu}+1\right) B_{n-1,C} - \frac{\lambda}{C\mu} B_{n-2,C} - \frac{\beta}{C\mu} B_{n-1+C,0}, \quad n = \overline{2, r-C-1},$$

$$A_{n+C,0} = \frac{\alpha}{\beta+\lambda} A_{nC} + \frac{\lambda}{\beta+\lambda} A_{n-1+C,0}, \quad n = \overline{2, r-C-1},$$

$$B_{n+C,0} = \frac{\alpha}{\beta+\lambda} B_{nC} + \frac{\lambda}{\beta+\lambda} B_{n-1+C,0}, \quad n = \overline{2, C-1},$$

$$A_{r-C,C} = \left(\frac{\lambda+\alpha}{C\mu}+1\right) A_{r-C-1,C} - \frac{\lambda}{C\mu} A_{r-C-2,C} - \frac{\beta}{C\mu} A_{r-1,0},$$

$$B_{r-C,C} = \left(\frac{\lambda+\alpha}{C\mu}+1\right) B_{r-C-1,C} - \frac{\lambda}{C\mu} B_{r-C-2,C} - \frac{\beta}{C\mu} B_{r-1,0},$$

$$A_{r0} = \frac{\alpha}{\beta} A_{r-C,C} + \frac{\lambda}{\beta} A_{r-1,0}, \quad B_{r0} = \frac{\alpha}{\beta} B_{r-C,C} + \frac{\lambda}{\beta} B_{r-1,0}.$$

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