Faculty of Computational
Mathematics and Cybernetics,
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XXIX International Seminar on Stability Problems for Stochastic Models

and

V International Workshop "Applied Problems in Theory of Probabilities and Mathematical Statistics related to modeling of information systems"

Book of Abstracts

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Survival Function Estimation in the Dependent Models of Random Censorship

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In survival analysis our interest focuses on a nonnegative random variables (r.v.-s) denoting death times of biological organisms or failure times of mechanical systems. A difficulty in the analysis of survival data is the possibility that the survival times can be subjected to random censoring by other nonnegative r.v.-s and therefore we observe incomplete data. There are various types of censoring mechanisms. In this article we consider only right censoring model and problem of estimation of survival function (s.f.) when the survival times and censoring times are dependent and propose new estimates of s.f. assuming that the dependence structure is described by a known copula function.

Let's consider$\{(X_k, Y_k), k \geq 1\}-$ a sequence of independent and identically distributed pairs of nonnegative r.v.-s with common joint s.f.$\overline{H}(x, y) = P(X_1 > x, Y_1 > y)$,$(x, y) \in \overline{R}^{+2}$, $\overline{R}^{+2} = [0, \infty] \times [0, \infty]$. We suppose that the marginal s.f-s $S^X(x) = P(X_1 > x)$ and $S^Y(y) = P(Y_1 > y)$ are continuous and $S^X(0) = S^Y(0) = 1$. Consider statistical model in which r.v.-s of interest (survival times) $\{X_k, k \geq 1\}$ are censored on the right by r.v.-s $\{Y_k, k \geq 1\}$ and at n-th stage of the experiment the observation available the sample $\mathbb{V}^{(n)} = \{(Z_k, \delta_k), 1 \leq k \leq n\}$, where $Z_k = min(X_k, Y_k)$,$\delta_k = I(Z_k = X_k)$ and $I(A)$ is the indicator of the event A . The problem is consist in estimating of the s.f. S^X by the sample $\mathbb{V}^{(n)}$. Note that according to Sclar's theorem jointly s.f. $\overline{H}$ can be expressed through the appropriate survival copula function $\overline{C}$ as $\overline{H}(x, y) = \overline{C}(S^X(x), S^Y(y)), (x, y) \in \overline{R}^{+2}$. In the sequel we assume that $\overline{C}$ is Archimedean copula, i.e. $\overline{C}(u, v) = \varphi^{[-1]}[\varphi(u) + \varphi(v)]$,$(u, v) \in [0, 1]^2$, where $\varphi : [0, 1] \to \overline{R}^+$ is some known generator function with the pseudo inverse $\varphi^{[-1]}$. In this article for the s.f. S^X we consider a new estimator of the form

$$S_n^X = \varphi^{[-1]}[\varphi(S_n^Z(x))^{\frac{(-\int_0^x I(\mathbb{J}_n(t)>0)\varphi'(\frac{\mathbb{J}_n(t)}{n})d\overline{\mathbb{N}}_n(t))}{(-\int_0^x I(\mathbb{J}_n(t)>0)\varphi'(\frac{\mathbb{J}_n(t)}{n})d\overline{\mathbb{N}}_n^Z(t))}}],$$

where

$$S_n^Z(x) = \frac{1}{n}\sum_{k=1}^n I(Z_k > x), \mathbb{J}_n(x) = nS_n^Z(x-),$$
$$\varphi(S_n^Z(x)) = -\int_0^x I(\mathbb{J}_n(t) > 0)[\varphi(\tfrac{\mathbb{J}_n(t)}{n}) - \varphi(\tfrac{\mathbb{J}_n(t)}{n} - \tfrac{1}{n})]d\overline{\mathbb{N}}_n^Z(t),$$
$$\overline{\mathbb{N}}_n(x) = \sum_{k=1}^n I(Z_k \leq x, \delta_k = 1), \overline{\mathbb{N}}_n^Z(x) = n(1 - S_n^Z(x)).$$

In some regularity conditions on s.f.-s S^X, S^Z and generator φ we prove the consistency and asymptotically normality properties of estimator S_n^X . We note that the estimator S_n^X is an extended variant of relative-risk power estimator (RRPE) of Abdushukurov [1-4] in independent censoring model. Note that RRPE have some good properties with respect to well known product-limit estimator of Kaplan-Meier (1958) and exponential-hazard estimator of Breslow (1972).

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Unit root test with dependent errors

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Introduction.

Following, Nelson and Plosser [4] the most popular classical unit root test has been the Dickey-Fuller test. The Dickey-Fuller statistic is traditionally obtained by estimating an autoregressive (AR) model by ordinary least squares estimation OLS. However, it is argued that the OLS estimator is non robust against additive outlier (AO). A test statistic based on this estimator might therefore also be non robust (more of details see Fellag [1]).So, the outlier sensitivity of the standard Dickey-Fuller statistic is caused by the non robustness of the OLS estimator.
Franses and Haldrup [2] studied effects of additive outliers on unit root Dickey-Fuller tests. They showed that there is over rejection of the unit root hypothesis when additive outliers occur. Also, Shin and al. [5]investigated effects of outliers on unit root tests in an AR(1) and more. They proved that the limiting distribution of the statistic of Dickey-Fuller is affected by an additive outlier. Also, they proposed a method to detect outliers and to adjust the observations. Maddala and Rao [3] show that, when n goes to infinity the impacts of finite additive outliers will go to zero.
In this work, the one sided unit root test of a first autoregressive model in the presence of an additive outlier is considered. We study the behavior of the

size and the power of the test when an additive outlier (AO) occurs at time k. And then we consider an innovation outlier (IO). However, in the two cases, we consider that the errors are not independent identically distributed (iid).

The model.

Consider a time series $\{x_t\}$ which follows the model

$$(1-\rho B)x_t = \epsilon_t \qquad t = \dots, -1, 0, 1, \dots, n$$

where $\{\epsilon_t\}_{t=1,\dots,n}$ is an autoregressive process of order one satisfying

$$(1-\phi B)\epsilon_t = \eta_t \qquad t = \dots, -1, 0, 1, \dots, n$$

$\{\eta_t\}_{t=1,\dots,n}$ is a sequence of independent normally distributed random variables with mean zero and variance 1 and B denotes the backshift operator such that $Bx_t = x_{t-1}$. We assume that $x_0 = 0$.

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Change-point problems with Bayesian approach

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The problem of the Bayesian estimation of the change-point in independent Gaussian samples is considered. The method of computation of the mode of the posterior density is investigated. The impact of an outlier on the performance of the Bayesian procedure is studied. Finally, two examples are given at the end of this paper to illustrate the method proposed.

key words: Bayesian estimation, Change-point, Gaussian models, Outlier.

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On some problems related to the non-symmetric Laplace distribution

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Here we give an example of a rather simple limit scheme which can be used as a justification of the asymmetric Laplace distribution as an asymptotic approximation.

Let a_1 and a_2 be two finite positive numbers. We say that a random variable (r.v.) X has the *asymmetric Laplace distribution*, if its distribution function has the form

$$G_X(x) = \begin{cases} \dfrac{a_1}{a_1+a_2} \cdot e^{a_2 x}, & x \le 0 \\ 1 - \dfrac{a_2}{a_1+a_2} \cdot e^{-a_1 x}, & x > 0. \end{cases} \tag{1}$$

It is easy to see that the density $p_X(x)$ corresponding to the distribution function $G_X(x)$ has the form

$$p_X(x) = \begin{cases} \dfrac{a_1 a_2}{a_1+a_2} e^{a_2 x} & \text{for } x \leqslant 0 \\ \dfrac{a_1 a_2}{a_1+a_2} e^{-a_1 x} & \text{for } x > 0. \end{cases}$$

The asymmetric Laplace distribution is a popular and widely used model, see, e. g., [2], [1].

We show that this distribution appears as a limit in some limit theorems for sums of a random number of independent identically distributed (i.i.d.) random variables. Consider a double array $\{X_{n,j},\ j = 1, 2, \ldots\}_{n\geqslant 1}$ of row-wise (i. e., for each n) i.i.d. r.v.'s. For an integer nonnegative k denote

$$S_{n,k} = X_{n,1} + \ldots + X_{n,k} \quad (S_{n,0} \equiv 0).$$

Let $\{N_n\}$ be a sequence of integer-valued nonnegative r.v.'s. Assume that for each n the r.v.'s $N_n, X_{n,1}, X_{n,2}, \ldots$ are independent. The symbol $\Longrightarrow$ will denote weak convergence.

Theorem. *Assume that there exist numbers $\mu \in \mathbb{R}$, $\sigma^2 \in (0, \infty)$, $\lambda \in (0, \infty)$ and a sequence of natural numbers $\{k_n\}_{n\geqslant 1}$ such that*

$$S_{n,k_n} \Longrightarrow X \quad (n \to \infty), \tag{2}$$

and

$$\frac{N_n}{k_n} \Longrightarrow U \quad (n \to \infty), \tag{3}$$

where the r.v. X has the normal distribution with expectation μ and variance σ^2 and the r.v. U has the exponential distribution with parameter λ. Then

$$S_{n,N_n} \Longrightarrow Z \quad (n \to \infty), \tag{4}$$

where the r.v. Z has the asymmetric Laplace distribution (1) with parametrs

$$a_1 = \frac{1}{\sqrt{\mu^2 + 2\lambda\sigma^2} + \mu}, \quad a_2 = \frac{1}{\sqrt{\mu^2 + 2\lambda\sigma^2} - \mu}.$$

We also consider some generalizations and statistical problems related to the asymmetric Laplace distribution (1).

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Stability of Applied Stochastic Models

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In order to make decisions concerning the optimization of any system performance under incomplete information, first of all, it is necessary to choose an appropriate mathematical model (see, e.g., Bulinskaya [1]). The model must be stable. Thus, one has to study the model sensitivity to small parameters fluctuations and underlying processes perturbations. For this purpose it is possible to use the local or global sensitivity analysis (see, e.g., Bulinskaya [2] and references therein), as well as probability metrics technique (see, e.g., Zolotarev [3]). If the model is stable it can be employed to obtain the optimal control, otherwise another model is validated.

We consider a class of input-output models arising in such applications as insurance, finance, dams, inventory, biology and so on (see, e.g., Prabhu [4]) and establish their stability.

Then the following three-step algorithm is proposed for optimization under incomplete information:

1. Obtain the form of optimal control for any planning horizon under assumption of known parameters and proceses distributions.

2. Find asymptotically optimal stationary policy in the same conditions.

3. Estimate unknown parameters and distributions on the base of previous system observations and use them instead of exact ones.

This algorithm is implemented for several new dam models.

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Local Particles Numbers in Branching Random Walk on an Integer Lattice

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The model of branching random walk (BRW) on $\mathbb{Z}^d$ with a single source of branching has been investigated recently (see, e.g., Yarovaya [1], Vatutin and Topchii [2]). BRW on $\mathbb{Z}^d$ can be classified as supercritical, critical or subcritical in view of certain relation between producing offsprings at the source of branching and walking outside of it.

For a supercritical BRW on $\mathbb{Z}^d$, a limit theorem for the *total number* of particles, as well as that for the *number of particles at a fixed point* of the lattice, are established in Yarovaya [1] and they have similar forms for all $d \in \mathbb{N}$.

A new effect arises for a critical BRW on $\mathbb{Z}^d$. In this case the limit distribution of the *total number* of particles is found in Vatutin and Topchii [2] and it turns out to be different for $d \leq 5$, $d = 6$ and $d \geq 7$. The limit theorems for the *number of particles at the source of branching* were also proved (see Bulinskaya [3] and Hu, Vatutin and Topchii [4]). However, the limit distribution of the *number of particles at any fixed point* of the lattice (not only at the source of branching), for a critical BRW on $\mathbb{Z}^d$, was unknown. In this talk we provide the solution of this problem.

Moreover, we consider the mean number $m(t; x, y)$ of the particles at a point $y \neq \mathbf{0}$ ($\mathbf{0} \in \mathbb{Z}^d$) and the probability $Q(t; x, y)$ of non-degeneracy at y at time t, given the starting point x. We show that $m(t; x, y)$ has the same asymptotic behavior ($t \to \infty$) as $m(t; x, \mathbf{0})$ up to a specified constant factor which is not equal to 1 in general. However, $Q(t; x, y)$ is equivalent to $Q(t; x, \mathbf{0})$, as $t \to \infty$, for any $d \neq 2$. The case $d = 2$ is studied as well.

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On Newman's conjecture

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Analysis of asymptotical behavior of the (normalized) sums of random variables is the vast research domain of Probability Theory having numerous applications. The limit theorems established for independent summands form here the classical core. In this regard one can refer to the monographs Gnedenko and Kolmogorov [1], Ibragimov and Linnik [2], Petrov [3] and Zolotarev [4]; see also references therein.

Stochastic models described by means of families of dependent random variables arose at the beginning of the last century. Thus the Gaussian and

Markov processes, martingales, solutions of the stochastic differential equations, mixing processes appeared as well as other important classes. Moreover, much attention has been paid to studying of random fields.

Since the 1960s due to the problems of mathematical statistics, reliability theory, percolation and statistical physics there arose the stochastic models based on the families of variables possessing various forms of positive or negative dependence (see, e.g., Bulinski and Shashkin [5]). The key role in these models belongs to the notion of association (in statistical physics the well-known FKG-inequalities imply the association).

We consider a random field, defined on an integer-valued d-dimensional lattice $\mathbb{Z}^d$, with covariance function satisfying a condition more general than summability. Such condition appeared in the well-known Newman's conjecture (see Newman [6]) concerning the central limit theorem (CLT) for stationary associated random fields. As was demonstrated by Herrndorf [7] and Shashkin [8], the conjecture fails already for $d = 1$. In the present talk, we show the validity of modified conjecture leaving intact the mentioned condition on covariance function. Thus we establish (see Bulinski [9]), for any integer $d \geq 1$, a criterion of the CLT validity for a wider class of positively associated stationary fields. The uniform integrability for the squares of normalized partial sums, taken over growing parallelepipeds or cubes in $\mathbb{Z}^d$, is a crucial property in deriving their asymptotic normality. So our result extends the Lewis theorem (Lewis [10]) proved for sequences of random variables. A representation of variances of partial sums of a field using the slowly varying functions in several arguments is employed in essential way.

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On comparison of UMVUE and MLE risks on high order asymptotic expansions for one-parameter exponential family

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Uniformly minimum variance unbiased estimators (UMVUE) and maximum likelihood estimators (MLE) play all-important role in current statistical research (see, for example, Voinov and Nikulin [1]). But how to choose the best among them? The unbiassed choice of the approach for research estimator can be made by computing the precision either of the possible estimators. The author propose solution for some of such problems for high sample size in the case of one-parameter exponential family. The precision of estimators is defined by means of quadratic and absolute risk functions. Some of the such problems were pioneered by Hwang and Hu [2].

The observation model description. Here is $X_1, \ldots, X_n$ – repeated sample, which elements have the same distribution as observation random variable ξ. The distribution of the random variable ξ belong to exponential family, which determined with the following expression:

$$f_C[x; a] = \exp\{\Phi_1[a] \cdot T[x] + \Phi_2[a] + d[x]\}, \quad x \in G \subset \mathbf{R}. \tag{1}$$

Here $f_C[x; a]$ is a distribution function of ξ with respect to a measure $\mu[x]$ (either Lebesgue or counting measure); G is this distribution support; $d[x]$, $T[x]$, $\Phi_1[a]\,\Phi_2[a]$ be known Borel functions; $a = \mathbf{E}(T[\xi])$, $a \in \mathbf{A} \subset \mathbf{R}$ is a central parameter of given distribution. In addition $a\Phi_1'[a] + \Phi_2'[a] = 0$ and $\Phi_1'[a] > 0$ for all $a \in \mathbf{A}$.

Basic results. Let us denote $\check{G}[a|S_n]$ and $\widehat{G}[a|S_n]$, where $S_n = \sum_{i=1}^{n} T(X_i)$, corresponding the MLE and UMVUE of the given parametrical function $G[a]$. Model (1) is used in [2] to comparison this estimators in assumption, that estimator $\widehat{G}[a|S_n]$ have series expansion, which coefficients be a value of order $\mathbf{O}(1)$ almost everywhere with respect to the measure $\mu[x]$. In this case to receipt of asymptotic expansionis used the same approach as in paper [3]. Follow this approach under $S_n = s = n\,a + z\sqrt{n\mathbf{V}T(\xi)}$, $z = \mathbf{O}(\sqrt{2\alpha \ln n})$, $1 < \alpha < 1.5$ and $n \to \infty$ was receive the asymptotic expansion of 6th order of function

$$\widehat{G}[a|s] = G[a] + \frac{zG'[a]}{\sqrt{n\Phi_1'[a]}} + \frac{(z^2-1)G''[a]}{2n\Phi_1'[a]} + \sum_{i=3}^{6} \frac{c_i}{n^{i/2}} + \mathbf{o}\left(\frac{1}{n^3}\right), \qquad (2)$$

$$c_3 = \frac{zG''[a]\Phi_1''[a]}{2(\Phi_1'[a])^{5/2}} + \frac{(z^3-3z)G^{(3)}[a]}{6(\Phi_1'[a])^{3/2}}, \quad c_4 = -\frac{(z^2-1)G''[a]\Phi_1''[a]^2}{2(\Phi_1'[a])^4} +$$
$$+\frac{(3z^2-2)\Phi_1''[a]G^{(3)}[a]}{6\Phi_1'[a]^3} + + \frac{(z^2-1)G''[a]\Phi_1^{(3)}[a]}{4(\Phi_1'[a])^3} + \frac{(z^4-6z^2+3)G^{(4)}[a]}{24(\Phi_1'[a])^2}.$$

Coefficients c_5, c_6 at (2) have more bulky representation than c_3, c_4.

The similar expansions are receive as for unbiased estimator $\widehat{\mathbf{V}}\left[\widehat{G}[a|S_n]\right]$ of variance $\mathbf{V}\left[\widehat{G}[a|S_n]\right]$ as well for normalized error of unbiased estimator $\widehat{G}_*[a|S_n] = (\widehat{G}[a|S_n] - G[a])/\sqrt{\widehat{\mathbf{V}}\left[\widehat{G}[a|S_n]\right]}$.

These expansions are used for extract first two expansion terms of the following functionals:

$$\mathbf{E}\left(\widehat{G}[a|S_n] - G[a]\right)^2, \ \mathbf{E}\left(\widehat{\mathbf{V}}\left[\widehat{G}[a|S_n]\right] - \mathbf{V}\left[\widehat{G}[a|S_n]\right]\right)^2, \ \mathbf{E}\left(\widehat{G}_*[a|S_n] - Z_n\right)^2,$$
$$\mathbf{E}\left|\widehat{G}[a|S_n] - G[a]\right|, \ \mathbf{E}\left|\check{G}[a|S_n] - G[a]\right|,$$

as well some another.

Particularly, together with N.Fedoseeva the following result is determined:

$$\mathbf{E}\left|\widehat{G}[a|S_n] - G[a]\right| = 2\varphi[0]\left|G'[a]\right|\left(n\Phi_1'[a]\right)^{-1/2}\left(1 + \frac{c}{2n\,\Phi_1'[a]}\right) + \mathbf{o}\left(n^{-3/2}\right),$$
$$c = \frac{G''[a]\,\Phi_1''[a]}{2G'[a](\Phi_1'[a])^2} - \frac{G^{(3)}[a]}{3G'[a]} + \frac{\Phi_1^{(3)}[a]}{12\Phi_1'[a]} - \frac{1}{6}\left(\frac{\Phi_1''[a]}{\Phi_1'[a]}\right)^2 + \left(\frac{G''[a]}{2\,G'[a]}\right)^2,$$

where $\varphi[0] = \dfrac{1}{\sqrt{2\pi}}$.

Received asymptotic expansions of risk function further is used to determination of the asymptotic expected deficiency of the UMVUE to the MLE of the model (1).

All results have a representation both on central parameter a, and on canonical $\theta = \Phi_1[a]$ and natural parameters of the exponential distribution family.

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On some analogue of the generalized allocation scheme

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Let $\xi_1, \xi_2, \ldots, \xi_N$ be independent identically distributed nonnegative integer valued nondegenerate random variables. In the generalized allocation scheme introduced by V.F. Kolchin [1] random variables $\eta'_1, \ldots, \eta'_N$ are considered with joint distribution

$$P\{\eta'_1 = k_1, \ldots, \eta'_N = k_N\} = P\Big\{\xi_1 = k_1, \ldots, \xi_N = k_N \;\Big|\; \sum\nolimits_{i=1}^{N} \xi_i = n\Big\}.$$

This scheme contains several interesting particular cases such as the usual allocation scheme and random forests.

In this paper we will study random variables $\eta_1, \ldots, \eta_N$ with joint distribution

$$P\{\eta_1 = k_1, \ldots, \eta_N = k_N\} = P\Big\{\xi_1 = k_1, \ldots, \xi_N = k_N \;\Big|\; \sum\nolimits_{i=1}^{N} \xi_i \leqslant n\Big\}.$$

It can be considered as a general allocation scheme when we place at most n balls into N boxes. In this general allocation scheme the random variable $\mu_{nN} = \sum_{i=1}^{N} I_{\{\eta_i = r\}}$ can be considered as the number of boxes containing r balls.

We study laws of large numbers, i.e. the convergence of the average $\frac{1}{N}\mu_{nN}$, as $n, N \to \infty$. We prove local limit theorems, i.e. we study the asymptotic behaviour of $P\{\mu_{nN} = k\}$. We obtain weak limit theorems for the maximum, i.e. we shall consider the asymptotic behaviour of $P\{\max_{1\leqslant i\leqslant N} \eta_i \leqslant r\}$.

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CLT for Associated Systems

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The purpose of this work is to establish a central limit theorem for stationary random fields which can be represented in terms of certain functions of quasi-associated random fields. We employ the following

Definition ([1]). A family of square integrable random variables $Y = \{Y_t,\ t \in T\}$ is called quasi-associated ($Y \in QA$) if for any disjoint finite sets $I, J \subset T$ and for any Lipschitz functions $F : R^{|I|} \to R$ and $G : R^{|J|} \to R$ one has

$$|cov(F(Y_i, i \in I), G(Y_j, j \in J))| \leqslant \sum_{i\in I}\sum_{j\in J} Lip_i(F) Lip_j(G) |cov(Y_i, Y_j)|.$$

Here Lip_i is the Lipschitz constant with respect to i-th coordinate. Namely, for a Lipschitz function $F : R^n \to R$ we put

$$Lip_i(F) = \sup_{x\in R^n} \sup_{\Delta x_i \neq 0} \frac{1}{|\Delta x_i|} |F(x_1, \ldots, x_{i-1}, x_i + \Delta x_i, x_{i+1}, \ldots, x_n) - F(x)|.$$

It is known ([2]) that every positively or negatively associated square integrable random field is also quasi-associated. Any Gaussian random field is necessarily quasi-associated (see [3]).

We consider a strictly stationary random field $Y = \{Y_k,\ k \in Z^d\}$, $d \in N$. Set $X_k = H(Y_k)$, $k \in Z^d$, where H is some Borel function. Our aim is to derive sufficient conditions for the random field $X = \{X_k,\ k \in Z^d\}$ to satisfy the central limit theorem, i.e. for the following convergence in distribution to hold for some $\sigma^2 < \infty$

$$\frac{1}{n^{d/2}} \sum_{k\in[1,n]^d} (X_k - \mathsf{E}X_k) \xrightarrow[n\to\infty]{d} N(0, \sigma^2).$$

Newman ([4]) obtained the central limit theorem for the case of H being a locally absolutely continuous function and Y being either associated or negatively associated. The method he employed requires some covariance estimates. Namely, it's assumed that the series

$$\sum_{k \in Z^d} cov(\tilde{H}(Y_0), \tilde{H}(Y_k)) \tag{1}$$

does converge for a certain nondecreasing function $\tilde{H}$ (which depends on H).

In this work we investigate convergence of such series and establish the central limit theorem for $Y \in QA$ and H being a locally Lipschitz or L_2 function. In order to estimate (1) we use the idea of Yu ([5]) and approximate the target function H with Lipschitz functions. This allows us to verify the convergence assuming that the distribution of Y_0 has light enough tails and the covariance function $r(k) = cov(Y_0, Y_k)$, $k \in Z^d$, decays rapidly enough as $|k| \to \infty$.

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Classification of patients' conditions in order to forecast outcomes of the treatment

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Background. The main aim of this research is the development of classification technique for patients who undergo Hyperbaric oxygenation procedure (HBO-procedure). This research has been performed in collaboration with the Khrunichev State Research and Production Space Center.

Method. Typical problem-solving method is the discriminant analysis which is classification technique for multivariate observations with training samples. Let there is m-dimensional attribute space ($m > 1$). Certain observations $x = \{x1, x2, ..., xm\}$ from X are points of this space. Discrimination task is to split set of multidimensional value realizations into p possible groups, denoted by R1, . . . , Rp, and to assign new observation to one of these groups. It is required to obtain set of crucial decision rules that enable to determine class Rk for arbitrary observation x from X. The analysis was performed with the use of Statistica 6.0.

Results. We observed a group of 161 patients. Three subgroups were identified empirically: HBO is well-tolerated and highly effective for the patients from the first subgroup; the patients with moderate effectiveness of the HBO-treatment were allocated to the second subgroup, and the patients without any effects or with deteriorating condition during procedure were allocated to the third subgroup. Each observation was assigned to one of three groups according to the expert estimations. For each patient cardiac physiological parameters were registered during each HBO-session. Exploratory data analysis was performed. We tested samples for normality employing Lillieforce modification of Smirnov-Kolmogorov criterion. For each group the results varied (see Table 1). Number of normally distributed variables declined with sample size increase. It should be noted that sample normality is not crucial for correct discrimination here. We identified training set consisting of most distinctive observations. In accordance to monitored data linear classification functions were plotted using stepwise discriminant analysis on basis of training set. It should be noted that three of our variables have appeared to be non-informative and were excluded from the final classification. The classification functions are:

$F1 = -222.787x1 + 0.077x2 - 1.023x3 + 18.464x4 - 785.4x5 + 0.109x6 + 0.244x7 + 43.387x8 + 0.099x9 - 0.074x10 + 1.826x11 - 0.042x12 - 21.876,$
$F2 = 185.201x1 + 0.114x2 + 0.560x3 - 9.669x4 + 43.946x5 - 1.010x6 + 0.237x7 + 224.283x8 - 0.035x9 - 0.131x10 + 1.732x11 - 0.017x12 - 19.402,$
$F3 = -42.333x1 + 0.187x2 + 1.754x3 - 20.396x4 - 278.369x5 + 0.312x6 + 0.408x7 + 270.880x8 - 0.023x9 - 0.247x10 + 3.760x11 - 0.098x12 - 43.832,$

where F1, F2, F3 are determined for good, satisfactory and ineffective groups respectively, x1-x12 are discriminant variables. Classifications functions

Good, n = 45									
x1	N	x4	N	x7	–	x10	–	x13	N
x2	–	x5	N	x8	N	x11	–	x14	–
x3	N	x6	N	x9	–	x12	–	x15	N
Satisfactory, n = 82									
x1	–	x4	N	x7	–	x10	N	x13	–
x2	–	x5	–	x8	–	x11	–	x14	–
x3	N	x6	–	x9	–	x12	–	x15	–
Ineffective, n = 13									
x1	N	x4	N	x7	–	x10	–	x13	N
x2	–	x5	N	x8	–	x11	N	x14	N
x3	N	x6	N	x9	N	x12	N	x15	–

Table 1: Test for normality

provide correct classification for each case in training sample. For test sample it was shown more the 75 % effectiveness (69.94, 80.43 and 76.92% for good, satisfactory and ineffective groups respectively).

Conclusions. Classification technique was designed according to statistical findings and expert assessment. We have analysed statistical significance of each discriminant variable and robustness of discriminant functions and classification procedure.

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Portfolio separation with α-stable, α-symmetric and pseudo-isotropic distributions

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Consider the portfolio choice problem for an agent in a frictionless financial market where the returns vector is $\mathbf{Y}$ plus some drift term – in addition there may or may not be a "risk free" investment opportunity. The celebrated *portfolio separation theorem* states conditions under which the market can be replaced by a smaller number of indices ("funds") without any welfare loss to the investors in question. We call this k-fund separation if k funds (possibly including the risk-free) suffice, and k-fund monetary separation if in addition there is a risk-free opportunity and this can always be chosen to be one of the funds.

The "Ross type" theorems (Ross [1]) concern the returns distributions for which the same k funds will do for any agent (typically assumed risk averse, in my contributions below assumed merely greedy but solvent). In a single period setting, it is known since Chamberlain [2] and Owen and Rabinovitch [3] that the elliptical distributions admit $\mathbf{Y}$ admit 2-fund separation (monetary if a risk-free opportunity exists), and it is known since Fama [4] that symmetric stable $\mathbf{Y}$ admit 2-fund monetary separation, provided there is a risk-free opportunity. The multi-period case will follow as a recursive application of the single-period model. In a continuous-time setting, the canonical model is the geometric Brownian motion, which by dynamic programming is easily seen to work as the Gaussian single period model. Khanna and Kulldorff [5] give a simple proof which does not rely upon dynamic programming.

For the single period, I will show – using fairly simple mathematics – that for all solvent agents:

1. Any pseudo-isotropic vector admits 2-fund monetary separation, provided that a risk-free investment exists. This also holds under "no short sale" conditions on the risky investments, or any cone conditions on these. (This case is near-trivial.)

2. The same holds for any skew α-stable vector, *provided* that there is a cone condition which has to restrict the skewness parameter to a singleton.

3. For the 1-stable case – possibly skew – one can remove the risk-free investment opportunity and have 1-fund separation (i.e., all investors choose the same portfolio, modulo scale).

4. For any *even* number k, any α-symmetric vector with $\alpha = k/(k-1)$ will admit k fund monetary separation, assuming *no* risk free opportunity. A similar result will fail – modulo gross degeneracies – for odd k.

5. Any skew-elliptical $\mathbf{Y}$ formed from conditioning k of the variables of an elliptical distribution, admits $k+2$ fund separation. This result is in press [6].

Possible extensions and ramifications will be discussed.

Simplifying the Khanna and Kulldorff approach, I will show for the continuous-time case that as long as the driving noise is a Lévy process $\mathbf{Y}(t)$, the properties of the single-period model are inherited (with $\mathbf{Y}(1)$ (necessarily infinitely divisible) for the above $\mathbf{Y}$).

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Second Order Approximations for Slightly Trimmed Sums

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Let $X_1, X_2, \ldots$ be a sequence of independent identically distributed real-valued random variables with common distribution function (*df*) F, and for each integer $n \geqslant 1$ let $X_{1:n} \leqslant \ldots \leqslant X_{n:n}$ denote the order statistics based on the sample $X_1, \ldots, X_n$.

Let k_n and m_n be sequences of integers such that $0 \leqslant k_n < n - m_n \leqslant n$, and $k_n \wedge m_n \to \infty$, as $n \to \infty$.

Put $\alpha(n) = k_n/n$, $\beta(n) = m_n/n$ and suppose that $\alpha(n) \vee \beta(n) \to 0$, as $n \to \infty$. Define the ν-th quantile of F by $\xi_\nu = F^{-1}(\nu) = \inf\{x : F(x) \geqslant \nu\}$, $0 < \nu < 1$, and let $W_i(n)$, denote X_i Winsorized outside of $(\xi_{\alpha(n)}, \xi_{1-\beta(n)}]$, i.e. $W_i(n) = \xi_{\alpha(n)} \vee (X_i \wedge \xi_{1-\beta(n)})$.

Consider a slightly trimmed sum given by

$$T_n = \frac{1}{n} \sum_{i=k_n+1}^{n-m_n} X_{i:n}.$$

Define

$$F_{T_n}(x) = P\left(\sigma_{W_{(n)}}^{-1} n^{1/2} \Big(T_n - \mu_n \Big) \leqslant x\right)$$

– the *df* of the normalized T_n, where

$$\mu_n = \int_{\alpha(n)}^{1-\beta(n)} F^{-1}(s)\, d\, s, \qquad \text{and} \qquad \sigma^2_{W_{(n)}} = Var(W_i(n)).$$

Suppose that F^{-1} is differentiable in $U = \big((0, \epsilon) \cup (1-\epsilon, 1)\big)$ for some $\epsilon > 0$. Let

$$\gamma_{3,W_{(n)}} = E(W_i(n) - \mu_{W_{(n)}})^3,$$

$$\delta_{2,W_{(n)}} = -\alpha^2(n) \frac{\big(\mu_{W_{(n)}} - \xi_{\alpha(n)}\big)^2}{f(\xi_{\alpha(n)})} + \beta^2(n) \frac{\big(\mu_{W_{(n)}} - \xi_{1-\beta(n)}\big)^2}{f(\xi_{1-\beta(n)})},$$

where $\mu_{W_{(n)}} = EW_i(n)$ and f denotes the density of F. Also introduce

$$\lambda_{1_{(n)}} = \frac{\gamma_{3,W_{(n)}}}{\sigma^3_{W_{(n)}}}, \qquad \lambda_{2_{(n)}} = \frac{\delta_{2,W_{(n)}}}{\sigma^3_{W_{(n)}}},$$

and for any real x define

$$G_n(x) = \Phi(x) - \frac{\varphi(x)}{6\sqrt{n}} \Big(\big(\lambda_{1_{(n)}} + 3\lambda_{2_{(n)}}\big)(x^2 - 1) + 6\sqrt{n} \frac{b_n}{\sigma_{W_{(n)}}} \Big),$$

where Φ is standard normal *df*, $\varphi = \Phi'$,

$$b_n = \frac{1}{2\sqrt{n}} \left(-\frac{\alpha(n)(1-\alpha(n))}{f(\xi_{\alpha(n)})} + \frac{\beta(n)(1-\beta(n))}{f(\xi_{1-\beta(n)})} \right),$$

b_n is a bias term (cf. [1]-[2]).

Let RV_ρ^∞ be a class of regularly varying in the infinity functions: $g \in RV_\rho^\infty \Leftrightarrow g(x) = |x|^\rho\, L(|x|)$, for $|x| > x_0$, with some $x_0 > 0$, $\rho \in R$, and $L(x)$ is a positive slowly varying function at infinity.

Suppose that $\limsup_{n\to\infty} \frac{n^s}{k_n \wedge m_n} < \infty$ for some $0 < s < 1$.

Here is our main result on asymptotic approximation of the Edgeworth type to a slightly trimmed sum for 'heavy' tailed F.

THEOREM. *Suppose that $f \in RV_\rho^\infty$, where $\rho = -(1+\gamma)$, $0 < \gamma < 2$, and assume that*

$$|f(x+\triangle x) - f(x)| = O\Big(f(x) \Big| \frac{\triangle x}{x} \Big|\Big),$$

whenever $\triangle x = o(|x|)$, as $|x| \to \infty$.

Then $|G_n(x) - \Phi(x)| \asymp (k_n \wedge m_n)^{-1/2}$, $n \to \infty$, $x \in \mathbb{R}$. Furthermore,

$$\sup_{x\in R} |F_{T_n}(x) - G_n(x)| = O\left(\frac{(\log k_n)^{5/4}}{k_n^{3/4}} + \frac{(\log m_n)^{5/4}}{m_n^{3/4}}\right), \quad n \to \infty\,.$$

The proof is based on a $U-$statistic type approximation and also uses a version of Bahadur's type representation for intermediate sample quantiles [3].

The talk based on the joint work with Roelof Helmers (CWI, Amsterdam, The Netherlands).

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An Improvement of the Remainder Term Estimates in the Lyapunov Theorem under Diverse Moment Conditions

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For $\delta \in [0,1]$ denote $\mathcal{F}_{2+\delta}$ a class of all distribution functions (d.f.'s) $F(x)$ satisfying conditions

$$\int_{-\infty}^{+\infty} x\,dF(x) = 0, \quad \int_{-\infty}^{+\infty} |x|^{2+\delta}\,dF(x) < \infty.$$

Let $X_1, X_2, \ldots$ be a sequence of independent random variables with d.f.'s $F_1, F_2 \ldots \in \mathcal{F}_{2+\delta}$. Denote

$$\sigma_j^2 = \mathsf{E}X_j^2, \quad \beta_{2+\delta,j} = \mathsf{E}|X_j|^{2+\delta}, \quad j = 1,2,\ldots,n.$$

$$s_n^2 = \sum_{j=1}^n \sigma_j^2, \quad \ell_n = \frac{1}{s_n^{2+\delta}} \sum_{j=1}^n \beta_{2+\delta,j}, \quad \tau_n = \frac{1}{s_n^{2+\delta}} \sum_{j=1}^n \sigma_j^{2+\delta},$$

$$\overline{F}_n(x) = \mathsf{P}(X_1 + \ldots + X_n < xs_n) = F_1 * \ldots * F_n(xs_n),$$

$$\Delta_n = \Delta_n(F_1,\ldots,F_n) = \sup_x |\overline{F}_n(x) - \Phi(x)|, \quad n = 1,2,\ldots,$$

$\Phi(x)$ being the d.f. of the standard normal law.

As is known (see, e.g., (Petrov, 1972), (Petrov, 1987)), under above assumptions for all $n \geqslant 1$ and $F_1,\ldots,F_n \in \mathcal{F}_{2+\delta}$

$$\Delta_n \leqslant C_0(\delta) \cdot \ell_n, \tag{1}$$

where $C_0(\delta)$ depends only on δ. The upper bounds for $C_0(\delta)$, $0 < \delta < 1$ were obtained in (Tysiak, 1983). Here we present sharpened bounds for the constants $C_0(\delta)$ as well as in the spirit of the works (Korolev and Shevtsova, 2010a), (Korolev and Shevtsova, 2010b) we prove a moment inequality with an improved structure

$$\Delta_n \leqslant C_1(\delta) \cdot (\ell_n + \tau_n), \tag{2}$$

the values of $C_1(\delta)$ being considerably less than that of $C_0(\delta)$. By virtue of the Lyapunov inequality, $\tau_n \leqslant \ell_n$, thus, for large values of ℓ_n/τ_n estimate (2) is better than (1). Also, sharpened estimates for the constants $C_0(\delta)$, $C_1(\delta)$ in the i.i.d. case are presented.

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The Markov processes connected with weak generalized convolutions

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A probability measure $\mu \in \mathcal{P}$ is weakly stable if for all $a, b \in \mathbb{R}$ there exists a probability measure $\lambda \in \mathcal{P}$ such that

$$T_a\mu * T_b\mu = \mu \circ \lambda$$

where $T_a\mu(A) = \mu(A/a)$ for every Borel set A when $a \neq 0$, $T_0\mu = \delta_0$ and by $\circ$ we denote multiplicative convolution. The measure μ generates a binary operation called a weak generalized convolution.

We give a construction of discrete time Markov processes based on the weak generalized convolution, i.e. such random walks that their increments are independent and instead of summation of unit steps we take their cumulation in the weak stability sense. Theorem about asymptotical properties such objects will be showed. As an example of constructed processes we present random walk under the Kendall convolution having the following probability kernel :

$$\delta_x \otimes_{\mu_\alpha} \delta_1 = |x|^\alpha \pi_{2\alpha} + (1 - |x|^\alpha)\delta_1,$$

for $x \in [-1, 1]$ and $\alpha \in (0, 1]$ where $\pi_{2\alpha}$ is the Pareto distribution with the density $2\alpha y^{-2\alpha+1} I_{[1,\infty)}(y)$. Considering the random walks under the Kendall convolution we obtain a new class of heavy tailed distributions containing the Pareto distribution $\pi_{2\alpha}$. We present basic properties of constructed random walks. This describes a wide class of stochastic processes, for which the Bessel process discussed by Kingman was till now the only known example.

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Uniform interval estimates of survival function based on the result of random censored sample data observations

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The registers of pathology studies are obtained as a result of sample monitoring researches.These registers serve as a basis for further patients' survival analysis and often one have to deal with censored data. Under random censoring each person is examined for a random time period $\tau_i, i = \overline{1, N}$, i.e. as a tests' results we get values for random variables $\zeta_i = \min\limits_{i=\overline{1,N}} \{\xi_i, \tau_i\}$, as well as additionally we get an information on whether the death occurred during time of examination or not. The random variables which describe this information could be presented as indicators of occurrence of random events:

$$\delta_i = 1_{\xi_i \leqslant \tau_i} = \begin{cases} 1, & \xi_i \leqslant \tau_i; \\ 0, & \xi_i > \tau_i. \end{cases}$$

As an empirical estimate of distribution of unknown survival function $F_\xi(\cdot)$ one could consider PL-estimate, i.e. the Kaplan-Meyer estimate (see [1]). For this pointwise PL-estimate of empirical distribution function which is the consistent statistics from censored observations we build uniform interval estimate for accepted significance level using the martingale technique and generalize nonparametric Renyi's criterion, which is well known for complete samples' studies.

Denote the counting process [2] built for censored data $\{\zeta_i, \delta_i\}$, $i = \overline{1, N}$ by $K_N(t)$, i.e. $K_N(t) = \sum_{i=1}^N 1_{\xi_i \leqslant t \leqslant \tau_i} = \sum_{i=1}^N 1_{\zeta_i \leqslant t, \delta_i = 1}$. Left continuous modification of given process can be represented as: $R_N(t-) = \sum_{i=1}^N 1_{t \leqslant \xi_i \wedge \tau_i} = \sum_{i=1}^N 1_{\zeta_i \geqslant t}$.

Kaplan-Meyer estimate of the distribution function F_ξ can be represented by the counting process $K_N(t)$ and its continuous modification $R_N(t-)$ as $\widehat{F}_N(t) = 1 - \prod_{0 \leqslant s \leqslant t} \left(1 - \frac{\Delta K_N(s)}{R_N(s-)}\right)$ where $\Delta K_N(s)$ is the jump of process $K_N(\cdot)$ at time s. The process $\sqrt{N}(\widehat{F}_N(t) - F_\xi(t))$ considered on the interval $[0, t^+)$ converges weakly to Gaussian process with mean 0 and covariance function of form $V(s,t) = \Phi(s)\overline{F}_\xi(s)\overline{F}_\xi(t), s \leqslant t$, where $\overline{F}_\xi(t) = 1 - F_\xi(t)$. Consistent estimate for $\Phi(t), t \in [0, t^+)$ is given by equation

$$\widehat{\Phi}_N(t) = \int_0^t \left(\frac{1}{N} R_N(s)\right)^{-1} \mu(s) ds =$$

$$= \sum_{i=1}^{l} \frac{N}{N-i} \ln \frac{\overline{\widehat{F}}_N(\zeta_{i-1})}{\overline{\widehat{F}}_N(\zeta_i)} + \frac{N}{N-l} \frac{\overline{\widehat{F}}_N(\zeta_{l-1})}{\overline{\widehat{F}}_N(t)}. \tag{1}$$

As criterial statistics for constructing uniform confidence intervals of the distribution function F_ξ we consider the stochastic process $\eta_N(t) = \sqrt{N} \left| \frac{\overline{F}_N(t) - F_\xi(t)}{\overline{F}_\xi(t)} \right|$ for which

$$\lim_{N \to \infty} P\left\{\sup_{0 \leqslant t \leqslant \alpha} \eta_N(t) < u\right\} = P\left\{\sup_{0 \leqslant t \leqslant \alpha} |W(\Phi(t))| < u\right\}$$

holds and

$$\widehat{\Phi}_N(t) = \int_0^t \left(\frac{1}{N} R_N(s)\right)^{-1} \mu(s) ds =$$

$$\sum_{i=1}^{\max\{i: \zeta_{i-1} \leqslant t < \zeta_i\}} \frac{N}{N-i} \ln \frac{\overline{\widehat{F}}_N(\zeta_{i-1})}{\overline{\widehat{F}}_N(\zeta_i)} + \frac{N}{N-l} \frac{\overline{\widehat{F}}_N(\zeta_{l-1})}{\overline{\widehat{F}}_N(t)}$$

is the consistent estimate of $\Phi(t), t \in [0, t^+)$.

Asymptotic confidence interval for F_ξ with significance level $\alpha = 1 - p$ is uniform over the interval $[0, r]$ (where $r = \max\{\zeta_i$, i: $\delta_i = 1\}$) is given by the next equation:

$$F_{Lower/Upper} = 1 - \widehat{\overline{F}}_N(t) / \max\left\{0; \left(1 \pm u_p \sqrt{\frac{\widehat{\Phi}_N(r)}{N}}\right)\right\}, \tag{2}$$

where F_{Lower} and F_{Upper} are the lower (left) and upper (right) borders of confidence interval respectively and u_p is quantile of L-distribution defined as $L(u) = \frac{4}{\pi}\sum_{k=0}^{\infty}\frac{(-1)^k}{2k+1}e^{-\frac{(2k+1)^2\pi^2}{8u^2}}$.

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Fractional α-stable process with dependent increments and its application to network traffic modeling

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Since the beginning of the '90s, accurate traffic measurements carried out in different network scenarios highlighted that Internet traffic exhibits strong irregularities (*burstiness*) both in terms of extreme variability as well as long-term correlations. These features, which cannot be captured in a parsimonious way by traditional Markovian models, have a deep impact on network performance and led to the introduction in network traffic modeling of α-stable distribution and self-similar processes [4].

In this work, we consider a generalization of fractional Brownian motion, which is able to capture both the above-mentioned features of real traffic.

Let $(B_H(t), t \geq 0)$ be fractional Brownian motion with Hurst parameter H, $(L^1_\alpha(t), t \geq 0)$, $(L^2_\alpha(t), t \geq 0)$ be α-stable subordinators, $0 < \alpha \leq 1$, and B_H, L^1_α and L^2_α are independent. Consider the new process

$$X(t) := \begin{cases} B_H(L^1_\alpha(t)) & , \quad t \geq 0, \\ -B_H(L^2_\alpha(t)) & , \quad t < 0, \end{cases}$$

Using straightforward calculation it can be proved the following result.

Theorem 1. *The above process X is self-similar process with Hurst parameter $H_1 = H/\alpha$.*

Using the multiplication theorem for stable distribution (see Zolotarev [3], theorem 3.3.1) we can get the following result.

Theorem 2. *The above process X has β-stable distributions, where $\beta = \alpha/H$, whose increments are stationary, but dependent.*

Define the cumulative traffic (or arrival) process $A(t)$, i.e. the amount of total load produced by a source in the time interval $[0,t]$, $t > 0$, by

$$A(t) := mt + (\sigma m)^{1/\beta} X(t) \ ,$$

where $m > 0$ is the mean input rate, σ is the scale factor, X is the process defined above.

Consider a single server queue with constant service rate $r > 0$ and infinite buffer space, where input is the stable self-similar process defined above ($r > m$ for stability). The buffer occupancy $Q(t,r)$ at time $t \in R^1$ (queue size or queue length) can be written as

$$Q(t,r) = \sup_{s \le t}(A(t) - A(s) - r \cdot (t-s)) \ .$$

The process $Q(t,r), t \in R^1$, is stationary. So the most interesting for us is the following probability of overflow:

$$\varepsilon = P(Q(0,r) > b) = P\left(\sup_{\tau \ge 0}(A(\tau) - r \cdot \tau)\right) \ .$$

Using the technique elaborated in papers [1],[2] we can get the lower bound for the probability of buffer overflow.

Theorem 3. *An asymptotic lower bound for the overflow probability is given by*

$$\varepsilon = P(Q(0,r) > b) \ge C(H_1) \cdot \sigma \cdot \frac{r}{r-m} \cdot b^{-\frac{1-H_1}{H_1}} \ , b \to \infty \ .$$

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Multivariate generalized Cox processes

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Let $N(t) = (N_1(t), \ldots, N_m(t))$ be a multivariate Poisson process (with dependent components in general), $\{X_j = (X_{j1}, \ldots, X_{jm})\}$ be a sequence of i.i.d. random vectors woth finite second moments, $\Lambda(t) = (\Lambda_1(t), \ldots, \Lambda_m(t))$ be a multivariate random process such that: $\Lambda_k(0) = 0$, $\Lambda_k(t)$ has nondecreasing paths, $E(\Lambda_k(t)) = b_k \cdot t$, $Var(\Lambda_k(t)) = s_k^2 \cdot t$, $b_k > 0$, $s_k^2 > 0$ for all $k = \overline{1, m}$. The processes $(N(t), t \geq 0)$ and $(\Lambda(t), t \geq 0)$ are independent.

We consider the following variant of multivariate generalized Cox process: $C(t) = (C_1(t), \ldots, C_m(t))$:

$$C_k(t) := \sum_{j=1}^{N_k(\Lambda_k(t))} X_{jk} \ .$$

In our report we will propose a necessary and sufficient condition for convergence of the distribution of $C(t)$ with nonrandom centering and normalization to shift mixture of multivariate normal distribution as $t \to \infty$.

Our result is the analog of the result from [1].

Some applications to actuarial and financial mathematics are considered.

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Portfolio of options with dependent underlying assets

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One of the main problem in financial mathematics is the problem of price evalution of financial asset. Many papers are devoted to evalution of derivatives. The most interesting derivative is option. The classical result in this field is famous Black-Scholes formula. To prove it we need to use complicated methods from stochastic analysis. In paper [1] Cox, Ross and Rubinstein have proposed more simple binomial model (CRR-model). In framework of this model they have calculated the value of option. This model is very simple and suitable for simulation. The Black-Scholes formula can be derived from its analog in CRR-model (see [1],[3]). In these two models one dimensional case is considered where we have one option for one asset.

In our report we represent the analogous results in the case of options for several assets whose prices are generated by the process:

$$(S_1, \ldots, S_m) = \sum_{k=0}^{n} \varepsilon_k \ , \ n = 1, 2, \ldots \ ,$$

where ε_k are i.i.d.r.v. which takes values $i = (i_1, \ldots, i_k), i_l = 0$ *or* 1 with probabilities p_i. For the first time this model was proposed in paper [2]. Each component of this process has binomial distribution and these components are dependent.

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On the absolute constants in the Katz–Petrov–Osipov inequalities

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For an $n \in \mathbb{N}$ let $X_1, ..., X_n$ be independent random variables with $\mathsf{E}X_i = 0$ and $0 < \mathsf{E}X_i^2 \equiv \sigma_i^2 < \infty$, $i = 1, ..., n$. Denote $S_n = X_1 + ... + X_n$, $B_n^2 = \sigma_1^2 + ... + \sigma_n^2$. Let $\Phi(x)$ be the standard normal distribution function. Denote $\Delta_n(x) = |\mathsf{P}(S_n < xB_n) - \Phi(x)|$, $\Delta_n = \sup_x \Delta_n(x)$. Let $\mathcal{G}$ be the class of real-valued functions $g(x)$ of $x \in \mathbb{R}$ such that

- $g(x)$ is even;
- $g(x)$ is non-negative for all x and $g(x) > 0$ for $x > 0$;

- $g(x)$ does not decrease for $x > 0$;
- the function $x/g(x)$ does not decrease for $x > 0$.

In 1965 V. V. Petrov [9], generalizing a result of M. Katz (1963) [2] to the non-i.i.d. case, proved that if $\mathsf{E}X_i^2 g(X_i) < \infty$, $i = 1, ..., n$, for a $g \in \mathcal{G}$, then there exists a finite positive absolute constant C_1 such that

$$\Delta_n \leqslant \frac{C_1}{B_n^2 g(B_n)} \sum_{i=1}^n \mathsf{E}X_i^2 g(X_i). \tag{1}$$

In 1979 V. V. Petrov [10] proved a non-uniform analog of (1): under the same conditions there exists a finite positive absolute constant C_2 such that for any $x \in \mathbb{R}$

$$\Delta_n(x) \leqslant \frac{C_2}{B_n^2(1+|x|)^2 g\big(B_n(1+|x|)\big)} \sum_{i=1}^n \mathsf{E}X_i^2 g(X_i). \tag{2}$$

In particular, the function $g(x) = \min\{|x|,\, B_n\}$, $x \in \mathbb{R}$, is obviously in $\mathcal{G}$. In this case inequality (1) turns into

$$\Delta_n \leqslant C_1 \bigg(\frac{1}{B_n^2} \sum_{i=1}^n \mathsf{E}X_i^2 \mathbb{I}(|X_i| \geqslant B_n) + \frac{1}{B_n^3} \sum_{i=1}^n \mathsf{E}|X_i|^3 \mathbb{I}(|X_i| < B_n) \bigg) \tag{3}$$

provided $\mathsf{E}X_j^2 < \infty$, $j = 1, \ldots, n$. This inequality was proved in 1966 by L. V. Osipov [6]. L. Paditz (1980, 1984) [7], [8] showed that in (3) $C_1 < 4.77$. In 1986 he lowered this estimate to $C_1 < 3.51$. In 2001 Chen and Shao [1] re-proved inequality (3) with $C_1 = 4.1$.

In 1979 V. V. Petrov [10] proved a non-uniform analog of (3): provided $\mathsf{E}X_j^2 < \infty$, $j = 1, \ldots, n$, there exists a finite positive absolute constant C_4 such that for any $x \in \mathbb{R}$

$$\Delta_n(x) \leqslant C_4 \bigg(\frac{1}{B_n^2(1+|x|)^2} \sum_{i=1}^n \mathsf{E}X_i^2 \mathbb{I}(|X_i| \geqslant B_n) +$$

$$+ \frac{1}{B_n^3(1+|x|)^3} \sum_{i=1}^n \mathsf{E}|X_i|^3 \mathbb{I}(|X_i| < B_n) \bigg). \tag{4}$$

In 2001 Chen and Shao [1] re-proved (4) by another techniques. The best known upper estimate $C_4 \leqslant 76.17$ is due to Neammanee and Thongtha (2007) [5].

In this communication we improve the results of our work [4] as well as those of [5] and show that in (1) and (3)

$$0.5409 < C_1 \leqslant 2.0110$$

(also see [3]) and in (2) and (4)

$$C_j \leqslant 47.657, \quad j = 2,\, 4,$$

in the general case and

$$C_j \leqslant 39.317, \quad j = 2, 4,$$

in the i.i.d. case. Thus, in (1) and (2) the constants C_1 and C_2 are universal, that is, they do not depend on the specific form of the function $g \in \mathcal{G}$.

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Central limit theorem for risk estimate of vaguelette-wavelet signal decomposition

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Many problems of telecommunications, plasma physics, computer graphics and other applied areas involve indirect noisy measurements where one faces a linear inverse problem in the presence of noise. So we consider the following model:

$$X_i = (Kf)_i + \varepsilon_i, \tag{1}$$

where X_i are the observed data, K is some linear operator, f is the unknown signal we wish to estimate, and ε_i are independent normal variables with zero mean and variance equal to σ^2. We suppose that K is homogeneous with index α. That is $K[f(a(x-x_0))] = a^{-\alpha}(Kf)[a(x-x_0)]$ for each x_0 and every $a > 0$.

Nonlinear wavelet methods of signal processing are becoming more and more popular because of their ability to deal with non-stationarity and capture local singularities of the signal. One possibility is to use the following approximate signal decomposition (see [1]):

$$f = \langle Kf, \varphi_{0,0}\rangle K^{-1}\varphi_{0,0} + \sum_{j=0}^{J-1}\sum_{k=0}^{2^j-1} \beta_{j,k}\langle Kf, \psi_{j,k}\rangle u_{j,k}, \tag{2}$$

where $\varphi_{0,0}$ is a scaling function, $\{\psi_{j,k}\}$ is a wavelet basis generated by a certain mother wavelet ψ, and $\{u_{j,k}\}$ is a corresponding "vaguelette" basis, which is stable if K is homogeneous (see [2]). This kind of decomposition is called vaguelette-wavelet decomposition (see [1]).

To filter out the noise we use thresholding method with soft-thresholding function $\rho_{T_j}(x) = \mathrm{sgn}(x)\,(|x| - T_j)_+$, and obtain an estimate of the signal:

$$\hat{f} = Y^A_{0,0}K^{-1}\varphi_{0,0} + \sum_{j=0}^{J-1}\sum_{k=0}^{2^j-1} \beta_{j,k}\rho_{T_j}(Y^W_{j,k})u_{j,k}, \tag{3}$$

where $Y^A_{0,0}$ is a noisy approximation coefficient and $Y^W_{j,k}$ are noisy wavelet coefficients of the signal. Here we use individual threshold $T_j = \sqrt{2\ln 2^j}\sigma$ for each decomposition level j. This threshold is called "universal" (see [3]).

Risk (average mean squared error) of soft thresholding method is defined as

$$r_J = \sum_{j=0}^{J-1} \sum_{k=0}^{2^j-1} \beta_{j,k}^2 \mathrm{E}(2^{J/2}\langle Kf, \psi_{j,k}\rangle - \rho_{T_j}(Y_{j,k}^W))^2. \qquad (4)$$

This expression contains unknown values $\langle Kf, \psi_{j,k}\rangle$, so it cannot be calculated and has to be estimated. In [3] D. Donoho and I. Johnstone proposed to use SURE estimate

$$\hat{r}_J = \sum_{j=0}^{J-1} \sum_{k=0}^{2^j-1} \beta_{j,k}^2 R_{T_j}(Y_{j,k}^W), \qquad (5)$$

where $R_{T_j}(x) = (x^2 - \sigma^2)\mathrm{I}(|x| \leq T_j) + (\sigma^2 + T_j^2)\mathrm{I}(|x| > T_j)$. This estimate is unbiased, i.e. $\mathrm{E}\hat{r}_J = r_J$. We prove that under certain conditions it is also asymptotically normal. The following theorem holds.

Theorem. *Let K be a homogeneous linear operator with index $\alpha > 0$. Let mother wavelet ψ have sufficient number of vanishing moments and satisfy certain conditions, which ensure that basis $\{u_{j,k}\}$ is stable (see [2]). Let Kf have support in $[0,1]$ and be Lipschitz continuous of order $\gamma > (8\alpha + 2)^{-1}$. Then*

$$\frac{\hat{r}_J - r_J}{\sqrt{2\sigma^4\beta_{0,0}^4(2^{4\alpha+1}-1)^{-1}\; 2^{(2\alpha+1/2)J}}} \Longrightarrow N(0,1) \quad as \quad J \to \infty. \qquad (6)$$

In (6) we do not use traditional normalization which involves variance of $\hat{r}_J$, because this variance depends on the unknown values $\langle Kf, \psi_{j,k}\rangle$. Proposed normalization allows to construct asymptotic confidence intervals for r_J.

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Optimal arrangements of queueing systems on the line

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BACKGROUND. The main idea of this research is to provide the information about asymptotically (second-order) optimal arrangement of service centers in accordance with the average distance between call-point and service center for systems with FIFO service discipline. Such models of queuing systems are used studying real systems in which the service is made by object placed over a territory.

METHOD. We based our research on the property of the optimal arrangement of service centers. A specific feature of the class of systems under consideration is the necessity of using the information about location of serving devices and positions of the entering calls and their density distribution. We also assume that the density carrier of incoming call is the segment.

RESULTS. In the research process we've found optimal arrangement minimize the criterion the average distance between call-point and service center. The properties of optimal arrangements are described and algorithms for constructing asymptotically optimal arrangements are presented.

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On the stochastic processes under generalized convolution

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A random vector X is weakly stable if

$$\forall\, a, b \in \mathbb{R}\, \exists \theta \quad aX + bX' \stackrel{d}{=} x\theta,$$

where X' is an independent copy of X, X and θ are independent and $\stackrel{d}{=}$ denotes equality of distributions. If μ is the distribution of X and λ is the distribution of θ this condition can be written in the following way:

$$\forall\, a, b \in \mathbb{R}\, \exists \lambda_{a,b} \quad T_a\mu * T_b\mu = \mu \circ \lambda_{a,b},$$

where T_a is the rescaling operator and $\mu \circ \lambda_{a,b}$ is the operation on measures corresponding to the product of independent variables.

Each weakly stable distribution defines a generalized convolution $\otimes_\mu$ in the following:

$$\delta_a \otimes_\mu \delta_b = \lambda_{a,b}, \qquad \lambda_1 \otimes_\mu \lambda_2 = \int_{\mathbb{R}} \lambda_{a,b}\, \lambda_1(da)\lambda_2(db).$$

We consider stochastic process $\{X_t : t \geqslant 0\}$ for which there exists a family of probability measures $\lambda_{[s,t)}$, $0 \leqslant s \leqslant t$ such that
1) $\lambda_{[s,t)} \oplus_\mu \lambda_{[t,u)} = \lambda_{[s,u)}$ for each $s \leqslant t \leqslant u$;
2) X_t has distribution $\lambda_{[0,t)}$.
Basic properties and main examples of such processes will be given.

On the Bound of the Constant in the Berry-Esseen Inequality for Two-Point Distributions

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Let X, X_1, ... , X_n be a sequence of independent random variables having the same two-point distribution: $\mathbf{P}(X=a)=q$, $\mathbf{P}(X=d)=p$, where $p+q=1$, $a<0<d$, $\mathbf{E}X=0$, $\mathbf{E}X^2=1$. Without loss of generality we assume $0<p\leqslant 0.5$. Denote $\Phi(x)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-t^2/2}dt$,

$$\Delta_n(p) = \sup_x \left|\mathbf{P}\Big(\frac{1}{\sqrt{n}}\sum_{j=1}^{n} X_j < x\Big) - \Phi(x)\right|, \quad \beta(p) = \mathbf{E}|X|^3 \equiv \frac{p^2+q^2}{\sqrt{pq}}.$$

We have proved (see [1], [2]) the following

Theorem 1.

$$C_0 \equiv \sup_{n,\,p}\Big(\frac{\sqrt{n}}{\beta(p)}\,\Delta_n(p)\Big) < 0.4215. \tag{1}$$

Note that today the best upper bound of the absolute constant in the Berry–Esseen inequality in general case is 0.4784 [3], [4].

Remark that by the well known Esseen result [5], the following lower bound holds, $C_0 \geqslant C_E \equiv \frac{\sqrt{10}+3}{6\sqrt{2\pi}} = 0.409732\ldots$. Thus, the constant in the right–hand side of the inequality (1) differs from C_E approximately by 3%: $\frac{0.4215-C_E}{C_E} = 0.0287\ldots$.

Now some words about the proof of Theorem 1. We start with the following smoothing inequality,

$$\Delta_n(p) \leqslant \frac{1}{2\pi} \sup_x \left| \int_{-\infty}^{\infty} \frac{f^n(t) - e^{-nt^2/2}}{-it} \, \frac{\sin(t\varkappa)}{t\varkappa} \, e^{-itx} \, dt \right| + \frac{\varkappa}{\sqrt{2\pi n}}, \qquad (2)$$

where $f(t)$ is the characteristic function of X, $\varkappa = \frac{1}{2\sqrt{pq}}$.

In order to obtain an appropriate estimate of C_0, we first consider $n \geqslant 200$. Dividing the integral in (2) into three ones by the special way, and deriving fine estimates of these integrals, we get the following statement. Define the function $\mathcal{E}(p)$ by the equality $\mathcal{E}(p) = \frac{2-p}{3\sqrt{2\pi}\left[p^2+(1-p)^2\right]}$. Note that $\max\limits_{0\leqslant p\leqslant 0.5} \mathcal{E}(p) = C_E$, $\min\limits_{0\leqslant p\leqslant 0.5} \mathcal{E}(p) = \mathcal{E}(0) = \frac{2}{3\sqrt{2\pi}}$, $\mathcal{E}(0.5) = \frac{1}{\sqrt{2\pi}}$ (see fig. 1).

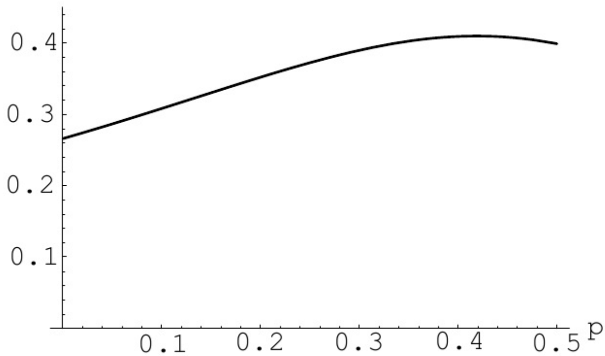

Figure 1: The function $\mathcal{E}(p)$.

Theorem 2. *Let* $\frac{4}{n} \leqslant p \leqslant 0.5$, $n \geqslant 200$. *Then*

$$\Delta_n(p) \leqslant \frac{\beta_3(p)}{\sqrt{n}} \Big(\mathcal{E}(p) + R(p,n)\Big),$$

where $R(p,n) \geqslant 0$ *satisfies the following properties: 1) for each* $0 < p \leqslant 0.5$, $R(p,n)$ *decreasing tends to 0 as* $n \to \infty$, *2)* $R(p,n) < 0.4215 - C_E$.

Further, using the inequality $\Delta_n(p) \leqslant \frac{0.33477}{\sqrt{n}}\Big(\beta(p) + 0.429\Big)$, proved in [4] for arbitrary i.i.d. random variables, we obtain in the case $0 < p \leqslant 0.02$ that $\frac{\sqrt{n}}{\beta(p)}\,\Delta_n(p) < 0.3582$.

As to the case $1 \leqslant n \leqslant 200$, we show by using a computer that $\max\limits_{1\leqslant n<200}\ \max\limits_{p\in(0,0.5]} \frac{\sqrt{n}}{\beta(p)}\,\Delta_n(p) < 0.4096 < C_E$.

Note that our method allows to obtain arbitrarily precise upper bounds of the type (1).

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A structural improvement of the non-uniform convergence rate estimates in the central limit theorem with applications to Poisson random sums

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For $n = 1, 2, \ldots$ let $X_1, \ldots, X_n$ be independent random variables with the common distribution function $F(x)$ and satisfying the conditions $\mathsf{E}X_1 = 0$, $\mathsf{E}X_1^2 = 1$ and $\beta_{2+\delta} \equiv \mathsf{E}|X_1|^{2+\delta} < \infty$ for some $0 < \delta \leqslant 1$. Denote the class of all distribution functions $F(x)$ of the random variable X_1 satisfying the above conditions by $\mathcal{F}_{2+\delta}$. Let $\Phi(x)$ be the standard normal distribution function. Denote $\Delta_n(x) = |\mathsf{P}(X_1 + \ldots + X_n < x\sqrt{n}) - \Phi(x)|$, $\quad x \in \mathbb{R},\ n \geqslant 1$. Then there exist positive finite numbers $A(\delta)$ such that

$$\sup_{x\in\mathbb{R}}(1 + |x|^{2+\delta})\Delta_n(x) \leqslant A(\delta)\beta_{2+\delta}/n^{\delta/2}. \tag{1}$$

Inequality (1) was proved by S. V. Nagaev (1965) for $\delta = 1$ and A. Bikelis (1966) for $0 < \delta \leqslant 1$ and not necessarily identically distributed random summands. The best known upper bounds for $A(\delta)$ are obtained by Y. Nefedova and I. Shevtsova (2011), in particular, $A(1) \leqslant 18.2$.

Generalizing the similar result of S. Gavrilenko to the case $0 < \delta \leqslant 1$, here we construct non-uniform estimates with the sharpened structure

$$\sup_{x\in\mathbb{R}}(1+|x|^{2+\delta})\Delta_n(x) \leqslant C(\delta)(\beta_{2+\delta}+1)/n^{\delta/2}, \quad n \geqslant 1,\ F \in \mathcal{F}_{2+\delta}, \qquad (2)$$

and describe an algorithm which allows to construct the upper bounds for the constants $C(\delta)$ for each $0 < \delta \leqslant 1$. For $\delta = 1$ this algorithm leads to the estimate $C(1) \leqslant 15.8$, which is substantially less than $A(1)$ in (1). Moreover, we construct a non-increasing function $C(\delta, x)$ of the argument $x \geqslant 0$ such that

$$\sup_{|t|\geqslant x}|t|^{2+\delta}\Delta_n(t) \leqslant C(\delta,x)(\beta_{2+\delta}+1)/n^{\delta/2}, \quad n \geqslant 1,\ F \in \mathcal{F}_{2+\delta}. \qquad (3)$$

and demonstrate that $\lim_{x\to\infty} C(\delta, x) = 1$ for all $0 < \delta \leqslant 1$.

Inequalities (2) and (3) are then applied to sharpening the constants in non-uniform estimates of the accuracy of the normal approximation to the distributions of Poisson random sums.

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Small Deviation Probabilities for Brownian Functionals

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We find exact small deviation asymptotics for some Brownian functionals beginning by the Brownian bridge B on $[0,1]$ in the weighted L_2-norm $\|\cdot\|_\psi$ for a large class of weights ψ.

Theorem 1. *Let the weight ψ, defined on $[0,1]$, be positive and twice continuously differentiable. Put $\vartheta = \int_0^1 \sqrt{\psi(t)}dt$. Then as $\varepsilon \to 0$*

$$\mathbb{P}\{\|B\|_\psi \le \varepsilon\} \sim \frac{2\sqrt{2}\psi^{1/8}(0)\psi^{1/8}(1)}{\sqrt{\pi\vartheta}} \exp\left(-\frac{\vartheta^2}{8}\varepsilon^{-2}\right).$$

Similar theorems are proved for Wiener process, Ornstein-Uhlenbeck process and some similar Gaussian processes. From this we deduce many exact small deviation results for integral functionals of weighted Bessel processes and bridges, Brownian local times, and related processes. Next theorem gives the exact small deviation asymptotics for the Brownian meander $\mathfrak{m}$ in the weighted quadratic norm.

Theorem 2. *Under conditions of Theorem 1, one has as $\varepsilon \to 0$*

$$\mathbb{P}\{\|\mathfrak{m}\|_\psi \leqslant \varepsilon\} \sim 4\sqrt{\frac{2}{3\pi}}\frac{\psi^{3/8}(0)}{\vartheta^{1/2}\psi^{1/8}(1)} \exp\left(-\frac{9\vartheta^2}{8}\varepsilon^{-2}\right).$$

This result is new even for the unit weight $\psi \equiv 1$.

Let $L_t^x(B)$ be the jointly continuous local time of a Brownian bridge B at the point $x \in \mathbb{R}$ up to time $t \in [0,1]$.

Theorem 3. *The following relation holds as $\varepsilon \to 0$:*

$$\mathbb{P}\left\{\int_{-\infty}^{\infty} (L_1^x(B))^3 dx \leqslant \varepsilon\right\} \sim \frac{8\sqrt{6}}{\sqrt{\pi}}\varepsilon^{-1} \exp\left(-\frac{9}{2}\varepsilon^{-1}\right).$$

This proposition refines on the result from [1], where the asymptotic relation was proved at the logarithmic level only. Many analogous results for other Brownian functionals can be found in [2].

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Local limit theorems for shock models

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In this paper we study the local behaviour of a characteristic of several types of shock models. In many physical systems, a failure occurs when the stress or the fatigue at time t, represented by $N(t)$, reaches a critical level E. We are interested in the time $\tau(E)$ for which this happens for the first time. In the cumulative shock model we assume that $N(n) = \sum_{i=1}^{n} X_i$ is an acumulation of independent shocks X_i. In the extreme shock model, we assume that $N(n) = X_{n:n}$ or $N(n,k) = X_{n-k+1:n}$ where the damage to the system is measured in terms of the largest shock up to now or by the k largest shocks. For these models we.obtain a local limit theorem for the corresponding time $\tau(.)$. We also discuss related models and present some multivariate extensions.

Keywords and phrases: Renewal theory, shock models, regular variation, extreme value theory, local limit theory.

AMS Subject Classification: 60F99; 60G40; 60K10; 26A12.

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On asymptotic tests of hypotheses for the shift/scale parameter ratio

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We consider an analogy of J.Neyman's asymptotic optimal $C(\alpha)$-test of hypotheses concerning the shift/scale parameter ratio based on randomly indexed central order statistics.

Let independent random values $Y_1, ..., Y_n$, N_n be given, $Y_1, ..., Y_n$ identically distributed with the common absolutely continuous distribution function $F((x-a)/b)$ depending on unknown shift and scale parameters a and $b > 0$ respectively, N_n takes integer non-negative values. Denote $c = a/b$. To test the null hypothesis $H_0 : c = c_0$ against the alternatives $H_1 : c = c_0 + \Delta/\sqrt{n}, \Delta > 0$, in [1] we constructed J.Neyman's asymptotic optimal $C(\alpha)$-test and its modification. On the multivariate case we generalize results of [3] where the Student's distribution acts as the limit distribution for some classes of statistics.The analogy of $C(\alpha)$-test we can use in situations where regularity conditions are not fulfilled and $C(\alpha)$-test does not exist.

Let $f(x) = F'(x)$, $0 < \lambda_1 < \lambda_2 < \lambda_3 < 1$, $F(\zeta_{\lambda_i}) = \lambda_i$, $0 < f(\zeta_{\lambda_i}) < \infty$, $i = 1, 2, 3$, N_n has a negative binomial distribution with parameters $(r, r/n)$, $r > 0$. We use $Y^{(N_n)}_{[\lambda N_n]+1}$ to denote an order statistic of the rank $[\lambda N_n]+1$ in the variational series $Y^{(N_n)}_1 \le ... \le Y^{(N_n)}_{N_n}$ constructed using the sample of the random size N_n,

$$T(Y) = Y^{(N_n)}_{[\lambda_2 N_n]+1}/(Y^{(N_n)}_{[\lambda_1 N_n]+1} - Y^{(N_n)}_{[\lambda_3 N_n]+1}).$$

The test has the critical region

$$W = \left\{ \frac{[T(Y)(\zeta_{\lambda_1} - \zeta_{\lambda_3}) - (\zeta_{\lambda_2} + c_0)]\sqrt{n}}{\sigma(c_0)} \ge t_{1-\alpha,2r} \right\},$$

$t_{p,k}$ denotes $p-$ quantile of Student's distribution with k degrees of freedom, $\sigma(c)$ is some function of c, λ_i, ζ_{λ_i}, $i = 1, 2, 3$. The asymptotic power of this test is given by

$$\mathbf{P}\{W|H_1\} = L_{2r}(t_{\alpha,2r} + \Delta/\sigma(c_0)),$$

where $L_{2r}(.)$ is the Student's distribution function with $2r$ degrees of freedom.

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The stability of the statistical model of the storm wind forecast over the territory of the West part of Russia and of the Baltic countries

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Advance forecast (from 12-36h) of strong summer winds that destroy buildings and electric wires makes possible to take proper measures and to reduce the losses and to protect the people. The existing graphic and calculation methods depend on subjective decisions of operators. Nowadays there is no hydrodynamic model for forecast of the maximal wind velocity, hence the main tools of objective forecast of these phenomena are statistical methods.

The meteorological situation involved the dangerous phenomena - the squalls and tornadoes and the wind with the velocity $V > 20m/s$ is submitted as the vector $X(A) = (x_1(A), x_2(A), , x_n(A))$, where n - the quantity of the empiric potential atmospheric parameters (predictors). Their values for the dates and towns, where are these phenomena, were accumulated in the set $\{X(A)\}$ - the teaching sample of the phenomena A presence. The teaching sample of the phenomena A absence or the phenomena B presence ($\{X(B)\}$) was obtained for such towns, where the atmosphere was instability and often the thunderstorms and the rainfalls were ($V < 8 - 10m/s$). The recognition model of the sets $\{X(A)\}$ and $\{X(B)\}$ was constructed with the help of the Byes approach.

Before the construction of the decisive rules of recognition we have solved the problem of choosing of the most informative and independent parameters. Thus for these phenomena the most informative predictors were selected without loosing information, those predictors being either representatives of blocks or independent informative predictors according certain criterion (the Mahalanobis distance and the Vapnik-Chervonenkis criterion of the minimum entropy $Hmin$). For this purpose the sample matrix $\mathbf{R}$ was corresponded to connected graph $\mathbf{G}$; 26 predictors are corresponded to the graph vertices, and the binary coefficients are corresponded to ribs of the graph $\mathbf{G}$. We have given the different thresholds of the connection $\mathbf{r}$. Then well to keep only the ribs in the graph $\mathbf{G}$, corresponding to the binary coefficient $r_{ij} \geq r$. The connected graph $\mathbf{G}$ breaks up to several connected subgraphs G_i in this case. Each subgraph Gi is corresponded to matrix $\mathbf{R}$ diagonal block of depend predictors.

Given optimal threshold $r = 0.5$ we obtained three blocks of dependent predictors and several isolated vertices, corresponding almost independent predictors. This way we have obtained the informative vector-predictor with the dimension $n = 6$. The assessments of this statistical model forecast were more better than synoptic method forecast.

The new forecast (to 12-24-36h ahead) of strong winds was developed by the using of the new hydrodynamic-statistical model based on the prognostic output fields of the operative hemispheric model of Russia. Statistical decisive rules for the calculation of the discriminant functions were obtained by same Byes approach for the samples of wind phenomena with the velocities $V > 20m/s$ and $V > 25m/s$ (including squalls and tornadoes). For this purpose the teaching samples were included the values of forty physically substantiated potential predictors. Before the construction of the decisive rules of recognition we also have solved the problem to select the most informative and independent parameters by same mentioned method of choosing of the independent and informative predictors (with the help of the criterion of the Mahalanobis distance and of the Vapnik-Chervonenkis criterion of the minimum entropy). We obtained the dimension n of the vector-predictor equal 8. This forecast method was tested during three years and recommended for the using in the operative synoptic practice in the Departments on the Meteorology at the European part of Russia. The results of storm wind forecast of two classes ($V > 20m/s$, $V > 25m/s$) were obtained very high (the Pirsy assessments are $T = 0,52 - 0,76$).

During last five years we developed and tested the new hydrodynamic-statistical storm wind forecast using same statistical model but on the base of the output prognostic fields of the new hydrodynamic regional model of Hydrometcenter of Russia (for 12-24-36-48h ahead). The forecast results are very high too. Now we noted that the stability of the assessments of the storm wind forecast also observed in this case and so we can satisfy that the developed statistical model of the storm wind forecast is stabile. We submitted a lot of examples of the operative forecast of strong squalls and tornadoes at the West part of Russia: in Petersburg on 5.07.2002y., in Kaliningrad on 8.08.2005y., at the Petersburg area in 2010-11 years and other, at the territory of the Baltic countries during 2009-2011 years. This method is successful for Europe too. We hope to continue the work with this statistical model of storm wind forecast with the earliness 60-72h on the base of the new Russian semilagrangian hydrodynamic middle-forecast model.

On a Slow Server Problem: Solution and Applications

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For queueing system with heterogeneous servers (QSHS) there exists a problem of the service rule optimization in order to minimize the sojourn time of customers in the system. Theoretically the problem uses the theory of controllable queueing systems (see for example Rykov (1975), Kitaev, Rykov (1995) and Sennott (1999), and develops a new direction of this theory connected with study of qualitative properties of optimal policies, Rykov (1999) that is based on the study of optimizing function of a model and uses their sup- or supper-modularity properties, Topkis (1978). For applications knowledge of qualitative properties of optimal policies allows to really construct their. This allows to really use appropriate models in different applications, including up-to-date telecommunication systems, Pedro (2005), Vishnevsky, Semenova (2007).

Firstly the problem has been stated and considered by B.Krishnamoorthy (1963) for the system with two servers. It was shown that the optimal policy has some monotonicity properties in the sense that it demand to use a quick server constantly and switch on the slow server only after the queue length reach an some threshold level. Then also for two servers the problem has been considered more detailed by B.Hajek (1984) and W.Lin& P.R.Kumar (1984), where the monotonicity of optimal policy has been proved. G.Kool (1995) propose a simplified proof the same result also for two servers. R.Weber (1993) formulate a conjecture that the result is true in general case. Appropriate solution has been done by V.Rykov (2001), where the suggested condition of the optimal service rule stability has been omitted that leads to incomplete of the formal proof of the result. It was remarked in F.Verycourt & Y.P.Zhou (2006). Improvement proof for generalized case of the QSHS with respect to mean lost minimization has been done in V.Rykov&D.Efrosinin (2009). V.Rykov & D.Efrosinin (2003) numerically showed that the monotonicity property of optimal rule also holds for the QSHS with additional structure of penalties for the servers using and customers waiting. In D.Efrosinin&L.Bruer (2006) and D.Efrosinin (2008) also numerically it was shown that the monotonicity property of optimal service rule preserves also for retrial service systems and for the systems with PH-distributed inter-arrival times.

In the talk these results will be summarized and some new applications for telecommunication systems will be proposed.

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Increase of image recognition reliability based on statistical approach

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In this paper, half-tone image recognition problem is examined. Let a set of R half-tone images $X_r = \|x^r_{uv}\|$, $(u = \overline{1, U},\ v = \overline{1, V}, r = \overline{1, R})$ be specified. Here U and V are the image height and width, R is the number of images in the database. It is required to assign a new input image $X = \|x_{uv}\|$ to one of the R classes. Traditional approach in image recognition [1, 2] is based on optimization of some measure of similarity (distance) between images ρ and nearest-neighbour method

$$\nu = \underset{r=\overline{1,R}}{\operatorname{argmin}} \rho\left(X/X_r\right). \tag{1}$$

The situation becomes more complicated when we want to guarantee sufficient reliability of decision [1]. Even if the declared average error rate of applied criterion (1) is low, there is no guarantee that recognition result for concrete image X is correct. This sort of questions are especially accute many factors (distance to object, foreshortening, illumination, etc.) are varied. More often in practice reliability is raised at the expense of decision delivery refusal if

$$\rho\left(X/X_\nu\right) > \rho_1 = const. \tag{2}$$

In this paper the possibility to increase image classification reliability using statistical approach and decision choice based on comparison maximum of a-posterior probability of an accessory of input object to a class with certain threshold is investigated. It was proven that if Kullback-Leibler information discrimination [3] is used as a measure of images similarity (1), then the following criterion will be optimal in Bayesian terms

$$\frac{exp\{-U \cdot V \cdot \rho\left(X/X_\nu\right)\}}{\sum_{r=1}^{R} exp\{-U \cdot V \cdot \rho\left(X/X_r\right)\}} < p_0 = const. \tag{3}$$

In our experimental study we consider an application of criterion (3) and image model which is based on calculation of gradient orientation histograms, for a face recognition problem. In our experiments the most popular faces databases (FERET, ORL, Yale and Essex) were used. As preliminary image processing the OpenCV library was used to detect faces. Each face was processed with median filter with 3x3 pixels window sizes. The range of gradient orientation values change is broken on 8 equal parts (i.e. each part's range is $\pi/4$ radians). To overcome a problem of non-uniform illumination, images were divided into 144 (12x12) fragments. Experimental results summary is presented in Table 1.

	FERET	ORL	Yale	Essex
Exhaustive search probability error (1)	11.1%	3.5%	7.88%	0.89%
False-Reject Rate (FRR) of (2)	10.61%	3.0%	7.88%	0.81%
False-Accept Rate (FAR) of (2)	0.42%	0.25%	0.0%	0.0%
Sum of FAR and FRR (2)	11.03%	3.25%	7.88%	0.81%
Threshold ρ_1 optimal value for (2)	0.364	0.444	0.56	0.502
False-Reject Rate (FRR) of (3)	3.49%	1.0%	3.64%	0.65%
False-Accept Rate (FAR) of (3)	2.02%	0.75%	2.42%	0.08%
Sum of FAR and FRR (3)	5.51%	1.75%	6.06%	0.73%
Threshold p_0 optimal value for (3)	0.504	0.505	0.506	0.502

Table 1: Comparative analysis of criteria (1), (2) and (3)

Apparently from this table, it is impossible to pick up a threshold ρ_1 appropriate simultaneously for all used databasesfor traditional criterion (2). However criterion (3) could be used with the fixed threshold. In Table 2 we resulted error rates where $p_0 = 0.5038$.

	FERET	ORL	Yale	Essex
FRR	3.35%	1.5%	5.46%	0.65%
FAR	2.51%	0.5%	1.21%	0.41%
Sum of FAR and FRR	5.86%	2.0%	6.67%	1.06%

Table 2: Error rates for criterion (3) and $p_0 = 0.5038$

Based on these tables it is possible to draw the following conclusion. First, accuracy of criterion (1) in which there is no recognition refusal alternative is lower than the accuracy of criterion (2). Secondly, not less traditional criterion (2) doesn't provide the best accuracy. And, thirdly, for the offered criterion (3) it is possible to pick up threshold not dependent on a particular database. Thus the offered approach has shown the best results for popular faces databases.

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Central limit theorem for the Euler characteristics of Gaussian excursion sets

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Excursion sets of Gaussian random fields have attracted much attention in recent years due to their applications to modeling spatial structures. In particular they appear in mathematical tomography and astrophysics. It is natural to consider the probabilistic behavior of the Minkowski functionals (volumes, surface areas, Euler characteristics) of such excursion sets. Starting from the classical Kac formula for the expectation of upcrossings number, a well-elaborated theory of first and higher order moments of these functionals has been developed. One can refer to the book by Adler and Taylor [1] for a detailed account. Establishing the asymptotic normality of the corresponding properly normalized functionals is a more difficult problem, since they usually depend non-smoothly on the realizations of a random field. First central limit theorems were proved in 1970s for the level crossing number of a smooth Gaussian process. A powerful method of proving central limit theorems is based on Hermite polynomials and Itô-Wiener expansions, see the review by Kratz [2]. This abstract is devoted to the central limit theorem for Euler characteristics of an excursion set of a Gaussian random field, which is observed in observation windows growing to infinity in a regular way.

Given a C^2 function $f : \mathbb{R}^d \to \mathbb{R}$, denote by $\nabla f(x)$ and $\nabla^2 f(x)$ its gradient and Hessian matrix at point $x \in \mathbb{R}^d$ respectively. Recall that the index $ind(A)$ of a symmetric nondegenerate matrix A is the number of its negative eigenvalues.

Let $X = \{X_t, t \in \mathbb{R}^d\}$ be a stationary Gaussian random field with realizations which are C^2 with probability one. For a bounded measurable set $B \subset \mathbb{R}^d$ denote its Lebesgue measure by $|B|$. For $u \in \mathbb{R}$ and any measurable $B \subset \mathbb{R}^d$ define the excursion set

$$A_u(X, B) := \{s \in B : X_s \geq u\}.$$

Consider a closed block $U = [a_1, b_1] \times \ldots \times [a_d, b_d] \subset \mathbb{R}^d$. A sequence of blocks $\{U_n\}_{n\in\mathbb{N}},\ U_n = \prod_{i=1}^d [a_{i,n}, b_{i,n}]$, is said to grow to infinity in a regular way if $\min_{i=1,\ldots,d}(b_{i,n} - a_{i,n}) \to \infty,\ n \to \infty$.

It is known ([1, p. 208]) that conditions on X imply that the Euler characteristics $\mathcal{EC}(A_y(X, U))$, where U is a closed block, is well-defined. Let $R(t) = cov(X_0, X_t)$, $t \in \mathbb{R}^d$. Finally, let $\{H_k, k \in \mathbb{Z}_+^d\}$ be the d-parametric Hermite polynomials orthogonal family.

Set $Y_u(t) = I\{X_t \geq u\}(-1)^{ind \nabla^2(-X_t)}, t \in \mathbb{R}^d$.

Theorem 1. Suppose that X is a C^2, stationary and isotropic centered Gaussian field such that $\int_{\mathbb{R}^d} |R(t)|dt < \infty$. Assume that $\{U_n\}_{n\in\mathbb{N}}$ is a sequence of blocks growing to infinity in a regular way. Then for any $u \in \mathbb{R}$

$$\frac{\mathcal{EC}(A_u(X, U_n)) - \mathsf{E}\mathcal{EC}(A_u(X, U_n))}{\sqrt{|U_n|}} \to N(0, \sigma^2(X, u)) \tag{1}$$

as $n \to \infty$, here the asymptotic variance $\sigma^2(X, u)$ equals

$$\sum_{k,l \in \mathbb{Z}_+^d} \frac{H_k(0) H_l(0)}{2\pi k! l!} \int_{\mathbb{R}^d} cov(H_k(X_0) Y_u(0), H_l(X_s) Y_u(s)) ds \tag{2}$$

and $m! := m_1! \dots m_d!$ for $m \in \mathbb{Z}_+^d$.

Many possible extensions of Theorem 1 arise from generalizing possible index sets. We give two examples when these sets are not blocks.

Theorem 2. Suppose that X is as in Theorem 1. Let $M \subset \mathbb{R}^d$ be a stratified C^2 manifold of dimension d and $U_n = nM := \{x \in \mathbb{R}^d : n^{-1}x \in M\}$, $n \in \mathbb{N}$. Then (1)–(2) hold for any $u \in \mathbb{R}$.

Recall that a sequence of bounded measurable sets $\{U_n\}_{n \in \mathbb{N}}$ grows to infinity in the Van Hove sense if for any $\varepsilon > 0$ one has

$$\frac{|(\partial U_n)^{(\varepsilon)}|}{|U_n|} \to 0, \quad n \to \infty,$$

here $(\partial B)^{(\tau)}$ stands for the τ-neighborhood of the boundary of B in Euclidean metrics (in fact one usually uses an equivalent definition, see [3, Section 3.1.1]). Note that if a sequence of blocks grows to infinity in a regular way then the Van Hove sense growth holds also.

Theorem 3. Suppose that X is as in Theorem 1. Let $\{U_n\}_{n \in \mathbb{N}}$ be a sequence of closed convex sets having C^2 boundaries and growing to infinity in the Van Hove sense. Then (1)–(2) hold for any $u \in \mathbb{R}$.

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On the Asymptotic Behavior of the Remainder Term in the Lyapunov Theorem

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Denote by $\mathcal{F}_3$ a class of all distribution functions (d.f.'s) $F(x)$ satisfying conditions

$$\int_{-\infty}^{+\infty} x\,dF(x)=0,\qquad \int_{-\infty}^{+\infty} |x|^3\,dF(x)<\infty.$$

Let $X_1,X_2,\ldots$ be a sequence of independent random variables with d.f.'s $F_1,F_2,\ldots\in\mathcal{F}_3$. Denote

$$\sigma_j^2=\mathsf{E}X_j^2,\quad \beta_j^3=\mathsf{E}|X_j|^3,\quad j=1,2,\ldots,n.$$

$$s_n^2=\sum_{j=1}^n\sigma_j^2,\quad \ell_n=\frac{1}{s_n^3}\sum_{j=1}^n\beta_j^3,\quad \tau_n=\frac{1}{s_n^3}\sum_{j=1}^n\sigma_j^3,$$

$$\overline{F}_n(x)=\mathsf{P}(X_1+\ldots+X_n<xs_n)=F_1*\ldots*F_n(xs_n),$$

$$\Delta_n=\Delta_n(F_1,\ldots,F_n)=\sup_x|\overline{F}_n(x)-\Phi(x)|,\quad n=1,2,\ldots,$$

$\Phi(x)$ being the d.f. of the standard normal law.

Here we prove that for all $n\geqslant 1$ and $F_1,\ldots,F_n\in\mathcal{F}_3$

$$\Delta_n\leqslant\inf_{C\geqslant 2/(3\sqrt{2\pi})}(C\ell_n+K(C)\tau_n)+C'\ell_n^{7/6},\tag{1}$$

and in the i.i.d. case

$$\Delta_n\leqslant\inf_{C\geqslant 2/(3\sqrt{2\pi})}\left(C\frac{\beta_1^3}{\sigma_1^3\sqrt{n}}+\frac{K(C)}{\sqrt{n}}\right)+C''\ell_n^{3/2},\tag{2}$$

with an optimal function $K(C)$ given in the explicit form, and we also provide some concrete values of the constants C' and C'' which decrease as ℓ_n decreases. As it was shown in (Shevtsova, 2010), the value $2/(3\sqrt{2\pi})$ of the constant C in the first terms of (1) and (2) cannot be lowered. This result improves the known estimates due to Prawitz (1975), Bentkus (1991) and Chistyakov (2001).

We also present an analogous result for the case, when the absolute moments of the order only $2+\delta$ with $0<\delta<1$ are finite.

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Periodically Correlated Hilbertian Processes

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The Hilbertian autoregressive moving average (ARMA) processes generalizes the classical ARMA model to random elements with values in Hilbert spaces. These models were introduced by Bosq (1991), then studied by several authors, as Mourid (1993), Besse and Cardot (1996), Pumo (1999), Mas (2002, 2007), Horvath, Huskova and Kokoszka (2010). Periodically correlated (PC) processes in general and PC autoregressive models in particular have been widely used as underlying stochastic processes for certain phenomena with cyclic autocorrelations.
PC Hilbertian processes, of weak type, were introduced and studied by Soltani and Shishehbor (1998, 1999). These processes assume interesting time domain and spectral structures.

In this work, we consider PC ARMA Hilbertian processes of orders $p, q \geq 1$. We defined periodically correlated ARMA(p,q) Hilbertian processes (PCARMAH(p,q))) as follows:
A centered discrete time second order Hilbertian process $\mathcal{X} = \{X_n, n \in \mathbb{Z}\}$ is called PCARMAH(p,q) with period T, associated with $(\boldsymbol{\epsilon}, \boldsymbol{\rho}_1, \boldsymbol{\rho}_2, \cdots, \boldsymbol{\rho}_p, \ \boldsymbol{\beta}_1, \boldsymbol{\beta}_2, \cdots, \boldsymbol{\beta}_q)$ if it is periodically correlated and satisfies

$$\begin{aligned} X_n &= \rho_{1,n}(X_{n-1}) + \rho_{2,n}(X_{n-2}) + \cdots + \rho_{p,n}(X_{n-p}) + \epsilon_n \\ &\qquad + \beta_{1,n}(\epsilon_{n-1}) + \beta_{2,n}(\epsilon_{n-2}) + \cdots + \beta_{q,n}(\epsilon_{n-q}), \end{aligned}$$

where $\boldsymbol{\epsilon}_n = \{(\epsilon_{nT}, \cdots, \epsilon_{nT+T-1})', n \in \mathbb{Z}\}$ is a zero mean, strongly second order orthogonal process in $\mathbf{H}$, $\boldsymbol{\rho}_i = (\rho_{i,0}, \cdots, \rho_{i,T-1}), i = 1, \cdots, p$ $\boldsymbol{\beta}_j = (\beta_{j,0}, \cdots, \beta_{j,T-1}), j = 1, \cdots, q$; and for $i = 1, \cdots, p \ \ j = 1, \cdots, q$,

$\{\rho_{i,n}, n \in \mathbb{Z}\}$, $\{\beta_{j,n}, n \in \mathbb{Z}\}$ are T-periodic sequences in $\mathcal{L}(\mathbf{H})$, bounded linear operators on $\mathbf{H}$, with respect to n, with $\rho_{p,n}, \beta_{q,n} \neq 0$. Our studies on these processes involve existence, strong law of large numbers, central limit theorem and parameters estimation.

Analysis of $M^{[X]}|G|1|r$ queue with a resume level

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Hysteretic overload control is the main mechanism for preventing signaling overload in SIP (Session Initiation Protocol) servers. Those mechanisms are discussed in IETF documents and drafts [1, 2].

We consider a variant of hysteretic overload control with a so called resume level [3]. This mechanism works as follows: input of messages shut down once the number of messages hits the maximum pool size until it decreases to a certain value (resume level). Messages which arrive during shutdown are lost. In addition we assume that messages arrive in batches for better fitness of our model.

We consider a single-server queuing system with maximum pool size of r customers. Batches of customers arrive as a Poisson process with rate λ. Each batch contains k customers with probability l_k, j batches contain k customers with probability l_k^j. The distribution function and mean of a customer service time are given by $B(x)$ and b respectively. Whenever the queue size reaches r, the arrival process is shut down and resumes after the queue size gets reduced to the resume level $m(0 \leq m \leq k-2)$. Our aim is to find the equilibrium queue length distribution observed in a arbitrary instant.

Let us denote by $X(t)$ the number of customers in our system at time t. We define $t_n, n = 1, 2, \ldots$ as instants at which either (i) an idle period is ended, of (ii) a service is completed not resulting in a shutdown mode. The discrete-time process $\{X(t_n+), n = 1, 2, \ldots\}$ will be embedded Markov chain. We first find the equilibrium probability distribution $\{q_j, j = \overline{0, r}\}$ for $\{X(t_n+)\}$.

Let us introduce β_k probability that k batches arrive during a customer service time: $\beta_k = \int\limits_0^\infty e^{-\lambda x} \frac{(\lambda x)^k}{k!} dB(x)$. Thus the equilibrium equation set for probability distribution $\{q_j\}$ takes the form of

$$q_j = q_0 \sum_{i=0}^{j+1} l_i \sum_{k=0}^{j-i+1} l_{j-i+1}^k \beta_k + \sum_{i=1}^{min(j+1,r-1)} q_i \sum_{k=0}^{j-i+1} l_{j-i+1}^k \beta_k + \delta_{m+1,j} q_r, \quad j = \overline{0, r-1} \quad (1)$$

$$q_r = q_0 \sum_{i=0}^{\infty} l_i \sum_{j=r-i+1}^{\infty} \sum_{k=0}^{j} l_j^k \beta_k + \sum_{i=1}^{r-1} q_i \sum_{j=r-i+1}^{\infty} \sum_{k=0}^{j} l_j^k \beta_k. \qquad (2)$$

where $\delta_{i,j}$ is the Kronecker's delta. The system of equations (1,2) can be solved numerically.

Having $\{q_j\}$ distribution we may find the equilibrium probability distribution $\{p_j, j = \overline{0, r+1}\}$ for the original process $\{X(t)\}$ as described in [4]. The result is

$$p_0 = C^{-1} \frac{1}{\lambda} q_0; \qquad (3)$$

$$p_j = C^{-1} \left\{ q_0 \sum_{i=0}^{j} l_i \int_0^{\infty} [1 - B(x)] e^{-\lambda x} \left(\sum_{k=0}^{j-i} l_{j-i}^k \frac{(\lambda x)^k}{k!} \right) dx + \right.$$
$$\left. + \sum_{i=1}^{j} q_i \int_0^{\infty} [1 - B(x)] e^{-\lambda x} \left(\sum_{k=0}^{j-i} l_{j-i}^k \frac{(\lambda x)^k}{k!} \right) dx \right\}, \quad j = \overline{1, m+1}; \qquad (4)$$

$$p_j = C^{-1} \left\{ q_0 \sum_{i=0}^{j} l_i \int_0^{\infty} [1 - B(x)] e^{-\lambda x} \left(\sum_{k=0}^{j-i} l_{j-i}^k \frac{(\lambda x)^k}{k!} \right) dx + \right.$$
$$\left. + \sum_{i=1}^{min(j,r-1)} q_i \int_0^{\infty} [1 - B(x)] e^{-\lambda x} \left(\sum_{k=0}^{j-i} l_{j-i}^k \frac{(\lambda x)^k}{k!} \right) dx + q_r b \right\}, \quad j = \overline{m+2, r}; \qquad (5)$$

$$p_{r+1} = C^{-1} \left\{ q_0 \sum_{i=0}^{\infty} l_i \int_0^{\infty} [1 - B(x)] e^{-\lambda x} \left(\sum_{j=r-i+1}^{\infty} \sum_{k=0}^{j} l_j^k \frac{(\lambda x)^k}{k!} \right) dx + \right.$$
$$\left. + \sum_{i=1}^{r-1} q_i \int_0^{\infty} [1 - B(x)] e^{-\lambda x} \left(\sum_{j=r-i+1}^{\infty} \sum_{k=0}^{j} l_j^k \frac{(\lambda x)^k}{k!} \right) dx \right\}; \qquad (6)$$

and $C = (\frac{1}{\lambda} + b) q_0 + b(1 - q_0 - q_r) + (r - m - 1) b q_r$.

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Asymptotic formula for disconnection probability of graph on two dimensional manifold

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A problem of a calculation of a graph disconnection probability is considered in a lot of papers. For a graph with sufficiently small number of arcs in [1] accelerated algorithms are constructed. These algorithms showed good results in a comparison with Maple 11. In [2] this problem is solved using Monte-Carlo method and some specific combinatory indexes and formulas. But when a number of arcs increases this problem becomes much more complicated. So it is necessary to construct convenient asymptotic formulas for connectivity or disconnection probability of graph with high reliable arcs. In this paper such problem is solved for planar graphs or graphs arranged on two dimensional manifolds.

Consider unoriented and connected graph G with finite sets of nodes U and of arcs W. Denote $\mathcal{L}(u,v)$ the set of all cross sections in G which divide nodes $u,v \in U,\ u \neq v,\ \mathcal{L} = \bigcup\limits_{u \neq v} \mathcal{L}(u,v)$. Put $d(L)$ a number of arcs in cross section L and define $D(u,v) = \min(d(L):\ L \in \mathcal{L}(u,v)),\ \ D = \min_{u\neq v} D(u,v),\ \mathcal{L}_* = \{L \in \mathcal{L}:\ d(L) = D\}$, C is a number of cross sections from the set $\mathcal{L}_*$.

Theorem 1. Suppose that graph arcs $w \in W$ fail independently with the probability h then the probability P of the graph G disconnection satisfies the formula $P \sim Ch^D,\ h \to 0$.

So to calculate asymptotic of graph disconnection probability it is necessary to find the constants C, D. These calculations are based on a concept of a graph G arranged on connected and two dimensional smooth manifold without edge $\mathcal{T}$ [3, chapter 1]. Suppose that between two nodes of the graph G there is not more than two arcs and there are not arcs beginning and ending at the same node (loops). Arcs do not intersect and may have only common nodes. Each node and each arc belong to some cycle with more than two arcs and more than two nodes.

Call faces (or cells) areas $S_i,\ i = 0,\dots,m$, of the manifold $\mathcal{T}$ limited by its cycles minimal by the set theory inclusions. So faces may have common nodes,

common arcs but have not common internal points. Put two faces adjacent if there is their common arc. Each arc belongs to two faces (is adjacent to two faces). Denote by δS_i the face S_i boundary.
Suppose that faces $S_1, \ldots, S_m$ are bounded and call them internal. Then the face $S_0 = \mathcal{T} \setminus \bigcup_{i=1}^{m} S_i$ may be called external. The face S_0 may be unbounded if for example the manifold $\mathcal{T}$ is a plane. It may be bounded also if for example $\mathcal{T}$ is a sphere or a torus.
(A). Suppose that each two internal faces $S_i,\ S_j,\ 1 \le i < j \le m$, may have no more than single common arc.
Examples of graphs satisfied Condition **(A)** are connected aggregations of quadrates from rectangular lattice or connected aggregations of hexagons from hexagonal lattice.
Denote $A_{i,j}$ the set of arcs adjacent to faces $S_i, S_j,\ 0 \le i \ne j \le m$, and put $n_{i,j}$ a number of arcs in the set $A_{i,j}$. Designate $M_{i,j} = C^2_{n_{i,j}}$, if $n_{i,j} > 1$ and $M_{i,j} = 0$ if $n_{i,j} \le 1$. Define $N = \sum_{1 \le i \le m} M_{i,0},\ M = \sum_{0 \le i < j \le m} M_{i,j}$,
Theorem 2. Suppose that Condition **(A)** and the inequality $N > 0$ are true then $C = N,\ D = 2$.
An example of a graph satisfied Theorem 2conditions is integer rectangle.
Theorem 3. If $M > 0$ then the equalities $C = M,\ D = 2$ are true.
Denote U_3 the set of the graph G nodes which are connected with three arcs and put K_3 the number of elements in U_3.
Theorem 4. If $M = 0,\ K_3 > 0$ then $C = K_3,\ D = 3$.
Examples of graphs which satisfy Theorem 4conditions are the dodecahedron [3, hapter 4, Figure 4.2] and integer tube obtained by a gluing of a pair of opposite sides in an integer rectangle with a size $M \times N,\ M > 1,\ N > 1$.
Theorem 5. If $M = 0,\ K_3 = 0,\ K_4 > 0$ then $C = K_4,\ D = 4$. An example of a graph satisfies Theorem 5conditions is a graph arranged on two dimensional torus and obtained by a gluing of two pairs of opposite sides in an integer rectangle with a size $M \times N,\ M > 1,\ N > 1$.

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Linear plans of Polya's multidimensional random walks

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In this article results received for linear plans of Polya multidimensional random walks (Janardan and Patil [1], Lumelskiy [2]) are reviewed. In particular, for polynomial and multidimensional hypergeometrical random walks.

Linear plans — plans of the first appearance with a stop border G : $\sum_{m=0}^{N} a_m x_m = n$ for polynomial random walks have been considered in work of Kagan, Linnik and Rao [3] and are characterized as the elementary polynomial plans, not reduced to the binomial: known for binomial plans geometrical conditions of completeness become false.

Hyperplane G represents stop border of Polia's random walks first appearance plan Π^G on points with integer non-negative coordinates of space R^{N+1}, only if some restrictions are imposed on parameters $n, a_0, a_1, \ldots, a_N$. We will formulate them.

A1. $n \in N$.

A2. $a_m \in Z$ for all $m = 0, 1, \ldots, N$, and at least one m exists $(0 \leq m \leq N)$, for which $a_m > 0$.

A3. $\mathrm{GCD}(a_0, a_1, \ldots, a_N) = 1$.

Conditions A1 — A3 are minimal requirements, If all of them are true statements connected with properties of border G have sense.

In the mentioned work [3] the A. V. Malyshev's theorem is resulted. This theorem establishes within the limits of resulted restrictions a condition of geometrical isolation of polynomial linear plans:

A4. $a_m \leq 1$ for all $m = 0, 1, \ldots, N$, and there exists at least one m $(0 \leq m \leq N)$, for which $a_m = 1$.

Geometrical isolation here is understood as impossibility of multinomial walks' trajectories penetration through the border.

The first results for linear plans of Polia's multidimensional random walks have been received in work Lumelskiy and Lvovskiy [4] — obvious expression for probabilities of frontier points $\Gamma \in G$ achievement and a condition of asymptotic normalcy for these probabilities.

Unfortunately, in this article it was not possible to receive a condition of statistically distributed isolation of Polia's multidimensional random walks which could guarantee correctness of identity: $\sum_{\Gamma\in G} P^G(O,\Gamma) = 1$. In other words this condition present crossing trajectories of stop border walks for finite time with probability 1.

This condition managed to be received in work of Tsylova and Ekgauz [5]:

A5. $\sum_{m=0}^{N} a_m p_m > 0$.

In those work also is shown that Poisson's approximation conditions for considered plans show very strong restrictions on a littleness of parameter α value. At this restrictions Polia's multidimensional random walks practically degenerates in polynomial ($\alpha = 0$). Therefore instead of Poison's approximation conditions have been found convergence conditions to multidimensional α-generalized Poison's distribution (Tsylova [6]).

In work Lumelskiy and Lvovskiy [4] it was not possible to receive obvious expression for a matrix of covariances (dispersions and correlation factors) in the normal limiting theorem (as it is known, multidimensional normal distribution in an explicit form includes only a determinant of a matrix of covariances and elements of a return matrix to it). This matrix of covariances (and, hence, asymptotic expression for a matrix of covariances of distribution of the probabilities generated by the linear plan of multidimensional Polia's random walk) in work made ready for the press of authors of this article.

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On stochastic interpretation of fractional powers of operators

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It is known that fractional powers of closed operators can be interpreted in terms of one-sided α-stable densities [1-3]. On the base of this interpretation, the Monte Carlo algorithm is proposed to calculate the action of fractional powers of closed operators and to find numerical solutions of fractional equations. The corresponding theorem is formulated and proved. Applications of the algorithm are demonstrated on some physical examples.

Theorem. *Let the equi-continuous semigroup* $\{T_t;\ t \geq 0\}$ *of* C_0*-class be defined on a Banach space* F*. The infinitesimal generating operator* A *of the semigroup* $\{T_t\}$ *is defined as*

$$Af = \lim_{h\downarrow 0}\ h^{-1}(T_h - I)\ f$$

with domain

$$D(A) = \{f \in F; \lim_{h\downarrow 0}\ h^{-1}(T_h - I)\ f \quad \textit{exists in } F\}.$$

Then, the infinitesimal operators A *and*

$$A_\alpha\ x \equiv -(-A)^\alpha x, \qquad \forall x \in D(A), \qquad 0 < \alpha < 1,$$

generate the semigroups linked via relation

$$\hat{T}_t f = \int_0^\infty t^{-1/\alpha} g_+^{(\alpha)}(t^{-1/\alpha}\tau)\ T_\tau f\ d\tau \equiv \mathsf{E}\ T_{t^{1/\alpha}S_+(\alpha)} f, \tag{1}$$

where

$$S_+(\alpha) = \frac{\sin(\pi\alpha\gamma_1)\sin[\pi(1-\alpha)\gamma_1]^{1/\nu-1}}{[\sin(\pi\gamma_1)]^{1/\nu}\ [\ln\gamma_2]^{1/\nu-1}},$$

is the one-sided stable random variable [4], and $g_+^{(\alpha)}(\tau)$ *is their probability distribution function, the random variables* γ_1 *and* γ_2 *are uniformly distributed in (0,1).*

Example. In the theory of anomalous (non-Debye) relaxation, the fractional operator in the relaxation equation

$$[1 +_{-\infty} \mathsf{D}_t^\alpha]^\beta f(t) = \delta(t)$$

corresponds to the Havriliak-Negami frequency-domain response function [5]:

$$\tilde{f}(\omega) = [1 + (i\omega)^\alpha]^{-\beta}, \quad 0 < \alpha, \beta < 1.$$

According to Eq. (1), the semigroup generated by the infinitesimal operator $[1 +_{-\infty} \mathsf{D}_t^\alpha]^\beta$, can be found as

$$\widehat{T}_\tau\ f(t) = \mathsf{E}\ \exp\left(-\tau^{1/\beta}S_\beta\right) f\left(t - \left[\tau^{1/\beta}S_\beta\right]^{1/\alpha} S_\alpha\right).$$

The inverse operator can be calculated with the help of an averaging procedure

$$[1 +_{-\infty} \mathsf{D}_t^\alpha]^{-\beta} f(t) = \mathsf{E}\ \left[\beta S_\beta^{-\beta} E^{\beta-1} f\left(t - S_\alpha E^{1/\alpha}\right)\right],$$

where E is the exponentially distributed random variable with the unit mean value. We use this formula to find a solution of fractional relaxation equation for arbitrary prehistories of charging-discharging process.

We also consider applications of the algorithm to solutions of the spinless Salpeter equation, and the cosmic rays propagation equation containing fractional material derivative.

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Truncated fractional stable distributions and the correspondence principle in thermodynamics of nanosystems

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According to the correspondence principle any new theory pretending to more wide field of application than previous one must include the old theory as a limit case. So, relativistic mechanics turns into the Newtonian one in the limit of small speed. The Bohr correspondence principle is a postulate in quantum mechanics requiring coincidence of its results with results of classical theory in the limit of large quantum numbers. In the case of nanostatistics, it must describe macrosystems in the limit of large sizes. Thus, transition to the classical statistical physics must be scale phenomena. Note that both classes under concideration (fractional stable distributions (FSDs) $q(x;\alpha,\omega,\theta)$ and Tsallis distributions $p_{q,\beta}(x)$) include the Gaussian distribution as a limit case at the particular values of α,ω and q. These parameters are determined by features of the system and the process, but they do not depend on the system size.

As we demonstrated in this report, FSDs play a role of intermediate asymptotics towards the Gaussian distribution for CTRW process with waiting time and path length characterizing by distributions with truncated power law tails. For example in the case of truncated Lévy flights, from physical point of view, the number n can be interpreted as a size of a system, and transition from large n's to small n's as transition from macroscale to mesoscale (nano) scale. In some sense, this is demonstration of the correspondence principle: the theoretical results convert from classical form to special nanoscale form without any special corrections "by hand".

Distributions with truncated power law tails are quite widespread in nanodynamics. For example, on-intervals distributed according to such law had been observed in a signal of blinking quantum dot fluorescence by Shimizu and coauthors [1]. They consider this behavior as a temperature-dependent saturation effect that alters the long time tail of the distribution, the saturation arises due to a secondary mechanism that limits the maximum on-time duration of the QD. In the subrecoil laser cooling process, PDF of recycling times may be truncated due to optical friction forces that limit large values of momentum [2]. Authors [3] had demonstrated with the help of molecular dynamics simulation that the fast stick-slip diffusion of a nanocluster bound weakly to an atomically flat surface is a truncated Lévy walk.

FSD as asymptotic solution of the one-dimensional CTRW-model is a function of two variables, coordinate and time. Investigation of a crossover from non-Gaussian Lévy statistics to the Gaussian one with the help of FSD allows to follow this phenomena as the scale (time or size) effect. We demonstrated that fractionally stable statistics reduces to normal one at large times in the case of truncated power law tailed distribution of waiting times. The trajectory of truncated Lévy flights transforms its form from the Lévy type to the Brownian one with increasing of observable scale.

Thus, we obtained a statistical scheme with a natural crossover from normal macrostatistics to FSD nanostatistics due to transition from macro- to nanoscales. This is what we mean under the term *correspondence principle*.

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Noncommutative Brownian motions associated with positive cones

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We present a construction of noncommutative Brownian motions with bm-independent increments, and with multi-dimensional time parameter positive cones. The bm-independence is a generalization of Muraki's monotonic independence and Bożejko's boolean independence, which appear in noncommutative probability. The bm-independence is defined for noncommutative random variables indexed by partially ordered sets. The construction of bm-Brownian motions follows the idea of Hudson-Parthasarathy, who defined noncommutative Brownian motion as position operator (i.e. sum of creation and annihilation operators) on Fock space (bosonic, fermionic, free, et.c.) of the indicator functions of intervals. In our case the intervals are taken in various partially ordered set, in particular in a vector space with given positive cone. The examples of these are symmetric cones in Euclidian spaces, including the Lorentz light-cone and the positive definite hermitian matrices.

The talk is based on joint work [1] with Anna Kula from the Jagiellonian University (Kraków, Poland) and on description of noncommutative central limit theorems for symmetric cones [2].

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On the stability of characterizations of the normal law by an identical distribution property

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The present report is devoted to the estimation of the stability of characterization of the normal law by the property of identically distributed linear statistics

$$X = X_1, \qquad S = \sum_{i=1}^{n} b_i X_i,$$

where $X_1, X_2, ..., X_n$ are i.i.d. random variables and $b_1, b_2, ...b_n$ are real coefficients.

Such a characterization theorem is well known (see, for example, the monograph by Kagan, Linnik, Rao [1], Theorem 13.7.2). If X and S are identically distributed and $\sum b_j^2 = 1$, then $X_1, X_2, ..., X_n$ is a normal sample. It is important to emphasize that in the formulation of this characterization theorem any moment restrictions are absent.

Our purpose is to prove that F is close to the normal distribution in the Lévy metric whenever the distribution of the linear statistic $\sum b_j X_j$ is close to F.

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Necessary conditions in the law of large numbers for martingales

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Typically, necessary and sufficient conditions in classical limit theorems for independent summands have its natural counterparts for martingales. However, in the martingale case, the latter usually give only sufficient part of these theorems and no longer necessary ones. In the talk, we investigate the simplest possible case – the convergence in probability of martingales to zero.

A new method for data processing and its application

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In our work we investigated how water vapor distribution in the atmosphere changed over the past 11 years. Nowadays global climate changes are in area of many scientific researches. Energy of the atmosphere is contained mostly in water vapor. Study of water vapor field can help us to explain and predict cyclones and hurricanes formation.

Processes in atmosphere are complex, they are difficult to predict, so the use of modern methods of applied mathematics is especially important. The authors had conducted the research of 64800 time series of density of water vapor in points with given geographical coordinates. In our work we used wavelet analysis. It's quite new method, but most effective for non-stationary signal processing (see [1,2]).

Wavelet transform of function $f(x)$ defined as

$$W_\psi f(a,b) = \int\limits_{-\infty}^{\infty} f(x)\frac{1}{\sqrt{a}}\overline{\psi\left(\frac{x-b}{a}\right)}dx, \quad b \in R, \; a > 0.$$

In this work a wavelet transform calculated with wavelet ψ

$$\psi(t) = e^{2\pi i t}e^{-t^2/2}.$$

Wavelet transform had been chosen for the decomposition of the original signal due to its nonstationary nature. The most informative results had been received in the case of using wavelet Morlet. For every scale value in wavelet decomposition 2-D array the main frequency had been calculated for the appropriate number of factors. Having this values in every given geographic point "frequency maps" had been created for the Earth's surface. Some periodic phenomena and high daily activity in the global field of water vapor had been found out during given researches. Using "frequency maps" zones with different variability of water vapor density had been localized. These frequencies show us how large are fluctuations of waver vapor density in different points of the Earth. There is strong relation between frequency maps and atmospheric phenomena.

In our research we developed a mathematical part of work. But this work is related to various fields of earth sciences: geophysics, meteorology, and in this areas there are still many open questions.

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On the rate of convergence for nonstationary continuous-time Markov chains

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Let $X(t)$ be a nonstationary continuous-time Markov chain on finite state space $0, 1, \dots, r$. The sharp explicit bounds on the rate of convergence for nonstationary birth and death processes have been obtained in our previous papers, see Zeifman [1], Zeifman, Bening, Sokolov [2]. Here we consider essentially more general class of processes.

Let $p_{ij}(s,t) = Pr\{X(t) = j \,|X(s) = i\}$ be transition probabilities, and $p_i(t) = Pr\{X(t) = i\}$ be state probabilities for $X = X(t)$.

We suppose that all transition rates have the following form:

- $Pr\{X(t+h) = i+k \,|X(t) = i\} = \lambda_k(t)h + o(h)$,
- $Pr\{X(t+h) = i-k \,|X(t) = i\} = \mu_k(t)h + o(h)$,

  for any i, any $k > 0$ and any $t \geqslant 0$.

We suppose also that nonnegative intensity functions $\lambda_k(t)$ and $\mu_k(t)$ are locally integrable on $[0, \infty)$, and moreover $\lambda_{k+1}(t) \leqslant \lambda_k(t)$, $\mu_{k+1}(t) \leqslant \mu_k(t)$ for all k and any $t \geqslant 0$.

Then the probabilistic dynamics of the process is represented by the forward Kolmogorov differential system:

$$\frac{d\mathbf{p}}{dt} = A(t)\mathbf{p}(t),$$

where

$$A(t) = \begin{pmatrix} a_{00}(t) & \mu_1(t) & \mu_2(t) & \mu_3(t) & \mu_4(t) & \cdots & \mu_r(t) \\ \lambda_1(t) & a_{11}(t) & \mu_1(t) & \mu_2(t) & \mu_3(t) & \cdots & \mu_{r-1}(t) \\ \lambda_2(t) & \lambda_1(t) & a_{22}(t) & \mu_1(t) & \mu2(t) & \cdots & \mu_{r-2}(t) \\ \cdots & & & & & & \\ \lambda_r(t) & \lambda_{r-1}(t) & \lambda_{r-2}(t) & \cdots & \lambda_2(t) & \lambda_1(t) & a_{rr}(t) \end{pmatrix},$$

and the following equalities hold: $a_{ii}(t) = -\sum_{k=1}^{i} \mu_k(t) - \sum_{k=1}^{r-i} \lambda_{r-k}(t)$ for any t, i.

We denote by $\| \bullet \|$ the l_1-norm, i.e. $\|\mathbf{x}\| = \sum |x_i|$, for $\mathbf{x} = (x_0, x_1...)^T$ and $\|B\| = \sup_j \sum_i |b_{ij}|$ for $B = (b_{ij})_{i,j=0}^r$.

Let $E_k(t) = E\{X(t)\,|X(0) = k\}$ be the mean of the process at the moment t under initial condition $X(0) = k$.

Let $\{d_i\}$, $i = 1, \ldots, r$, be a sequence of positive numbers.

Put $d = \min_{1\leqslant i\leqslant r} d_i$, $G = \sum_{i=1}^r d_i$, $W = \min_k \frac{d_k}{k}$.

Consider the expressions

$$\alpha_i(t) = -a_{ii}(t)+\lambda_{r-i+1}(t)-\sum_{k=1}^{i-1}(\mu_{i-k}(t)-\mu_i(t))\frac{d_k}{d_i}-\sum_{k=1}^{r-i}(\lambda_k(t)-\lambda_{i+r-1}(t))\frac{d_{k+i}}{d_i},$$

and

$$\alpha(t) = \min_{1\leqslant i\leqslant r} \alpha_i(t).$$

Theorem. Let there exist a sequence $\{d_j\}$ of positive numbers such that

$$\int_0^\infty \alpha(t)\,dt = +\infty.$$

Then $X(t)$ is weakly ergodic and the following bound holds

$$\|\mathbf{p}^*(t) - \mathbf{p}^{**}(t)\| \leqslant \frac{8G}{d} e^{-\int_s^t \alpha(u)du},$$

for any initial conditions $\mathbf{p}^*(s), \mathbf{p}^{**}(s)$ and any $s, t, \quad 0 \leqslant s \leqslant t$.

Moreover, $X(t)$ has the limiting mean $\varphi(t)$, and

$$|E(t,k) - \varphi(t)| \leqslant \frac{4G}{W} e^{-\int_0^t \alpha(u)du},$$

for any k and $t \geqslant 0$.

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ASSOCIATED WORKSHOP
V International Workshop "Applied Problems in Theory of Probabilities and Mathematical Statistics related to modeling of information systems"
Autumn Session

Load Control Technique with Hysteresis in SIP Signaling Server

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The threshold-based load management [1, 3] is an essential tool in the prevention of various types of congestion in telecommunication networks [1-4]. One mechanism is the hysteretic control, which uses two types of thresholds to control congestion – congestion onset threshold and congestion abatement threshold. Criteria for the determination of SIP server congestion status are the number of messages in the queue for CPU service, i.e. buffer occupancy. When the buffer occupancy is increasing and it exceeds the threshold H, called onset congestion threshold, the congestion is determined. The incoming load should be reduced to avoid overloading. However, in order to avoid oscillations, the load does not return to normal load value immediately, but after a while, when the buffer occupancy is decreasing and it becomes below the threshold L, called a congestion abatement threshold. That technique is called hysteresis overload control.

The paper deals also with the study of SIP-server congestion control mechanisms. We detail the typical examples of detecting congestions, the problem of developing overload control mechanisms and requirements for them in accordance with the currently known IETF documents [4, 5]. We note that the mechanism, studied in papers [2, 3], uses hysteretic technique, however the term hysteresis congestion control is not mentioned. And no mathematical model is proposed in these papers to analyze the parameters and indicators of overload control.

Using the concept of hysteresis load control introduced for SS7 in [1], we construct a generic model of SIP-server with hysteresis load control technique in terms of queuing theory as in paper [6]. We consider a mathematical model, which takes into account two types of incoming flows of messages – INVITE

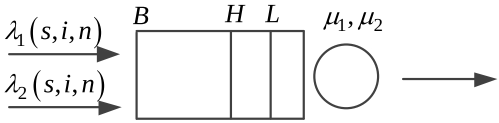

Figure 1: The $M_2|M_2|1|\langle L,H\rangle|B$ queuing model

and non-INVITE. This separation between two flows exists because all the mechanisms would rather drop INVITE messages than non-INVITE messages whenever overload occur. Two Poisson customer flows arrive at the system $M_2|M_2|1|\langle L,H\rangle|B$, shown in fig. 1, with the intensity $\lambda_1(s,i,n)$ and $\lambda_2(s,i,n)$ correspondingly as shown in fig. 2.

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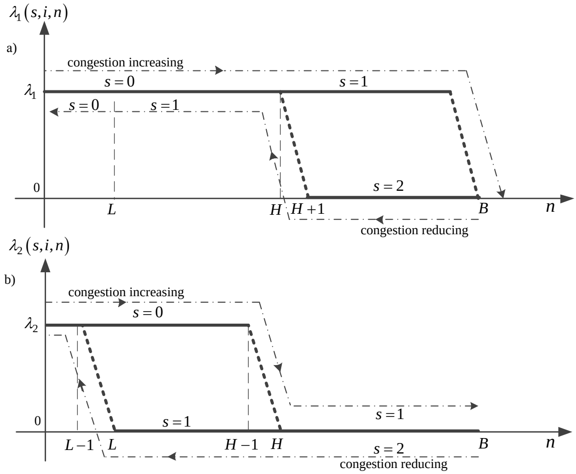

Figure 2: Hysteretic load control

System with different types of service resetting

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Introduction

In many queueing situations, a notable and inevitable phenomenon in the service facility is its breakdown and consequent repair. Nevertheless, in most of the queueing literature the server is always available, although this assumption is evidently unrealistic. Indeed, queueing systems with server breakdowns are very common in communication systems but an important distinguishing feature is to reset the service after a service interruption, that is, one speaks of preemptive resume (PR) if the interrupted customer can continue his service, of preemptive repeat identical (PRI) if his service is restarted or of preemptive repeat different (PRD) if his service is restarted with a new service time.

Developing analytical models to be used for analyzing their performance is a very important issue which has been dealt by several researchers. Most of the existing models focus on continuous-time models; however, works related to discrete-time systems with server interruptions can be found in Fiems and Bruneel [1]; Fiems, Steyaert, and Bruneel, [2]; Gaver [3]; Demoor and Fiems [4].

The main purpose of this work is to spread the queueing theory about unreliable servers with different types of service resetting after a service interruption to the discrete-time retrial queues. Hence, this paper deals with the study of a discrete-time retrial queue subject to active breakdowns, i.e., the server can fail only during the service period.

The mathematical model

In this paper we study a one line discrete-time queueing system with unreliable service. The customers arrive to the system according to a geometric distribution of parameter a. Every arrival to the system consists of a random number of packets. The number of packets of a customer is distributed by the law $\{b_i, \quad i \geqslant 1\}$, moreover every packet has the same length of T slots.

The server can only fail in a working period, moreover the failure can only be of one of the following different types:

The first type of failure, in a slot, occurs with probability θ_1. With this type of interruption, the server repairs itself in a random number of slots, distributed by the law $\{c_{1,i}, \quad i \geqslant 1\}$, and after the service is repaired the server continues servicing.

The probability of failure in the second type is θ_2, the server repairs itself in a random number of slots, distributed by the law $\{c_{2,i}, \quad i \geqslant 1\}$. However, the difference with the first type, is that after the completion of repairs, the server starts servicing from the beginning of the package that was in the server while the rejection occurred.

The third type of failure is characterized by the probability of rejection θ_3 and the server repairs itself in a random number of slots, distributed by the law $\{c_{3,i}, \quad i \geqslant 1\}$, moreover after the end of repairs the customer begin its service anew with the same number of packets that had before arriving to the server. In this case, if at any moment there is a failure of the third type then this moment will be considered as the moment of the arrival of the customer to the server until the next failure of fourth type occurs.

Finally, the fourth type of failure can occur with probability θ_4 where the distribution of the repair number of slots is $\{c_{4,i}, \quad i \geqslant 1\}$, after the completion of repairs the number of slots of the customer in service is developed anew with the initial distribution $\{b_i, \quad i \geqslant 1\}$.

It is obvious that $\theta = 1 - \sum_{i=1}^{4} \theta_i$ represents the probability that the server does not fail.

For the rest of the paper we will assume that the length of the slot on which it occurred the failure, of any type, it is joined with an appropriate repair time, i.e. the actual number of slots for the repair time will be no less than two.

The generating function $\psi(z)$ of the distribution of the service time of a customer with an initial random length is

$$\psi(z) = \frac{A_0(z)}{1 - A_1(z)}. \tag{1}$$

and

$$A_0(z) = \sum_{N=1}^{\infty} b_N \varphi_1^N(z) \left[1 - \varphi_3(z) \frac{1 - \varphi_1^N(z)}{1 - \varphi_1(z)}\right]^{-1},$$

$$A_1(z) = \sum_{N=1}^{\infty} b_N \varphi_4(z) \frac{1 - \varphi_1^N(z)}{1 - \varphi_1(z)} \left[1 - \varphi_3(z) \frac{1 - \varphi_1^N(z)}{1 - \varphi_1(z)}\right]^{-1},$$

where

$$\varphi_i(z) = \frac{f_i(z)}{1 - f_2(z)}, \quad i = 1, 3, 4,$$

$$f_1(z) = (\theta z + \theta_1 c_1(z))^T,$$

$$f_i(z) = \frac{\theta_i c_i(z)}{1 - \theta z - \theta_1 c_1(z)} \left[1 - (\theta z + \theta_1 c_1(z))^T\right], \quad i = \overline{2, 4}.$$

Equation (1) allows, with detail, to calculate the moments that a customers spends in the server.

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Probabilistic Approaches for Amplifying Error Correction of Computing Devices: State-of-the Art and Ways for Improvement

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In this talk we consider a problem of correct computations by an arithmetical logical unit (ALU), designed to compute an unknown function $f(x)$ of input values from the domain X, assuming that the device produces incorrect results $f_e(x) \neq f(x)$ for a small fraction of X. *Probabilistic self-correction* refers to the case where at least one random action is used in the operations, e.g., randomly chosen inputs and non-deterministic numbers of steps. In previous work we consider ways to increase (*"amplify"*) the probability of correct computation [1]. That is, to decrease the fraction of incorrect results, using random-reducibility properties of the functions finite fields [2]. We use a finite field as an input domain assuming that this is a result of discretization of real input signals. We demonstrate that the random reducibility-based self-correction approach, originally suggested to *amplify* the reliability of programs [2], can be used in the scope of non-finite fields for self-correcting hardware. For this end, it is necessary to provide a specific number of batches that yield sufficient probability for the majority of the batches to be correct; thereby enabling using majority vote procedures for self correction. We use error correction encoding (Reed-Solomon code [3]) for each input data batch leading to a reduction of the necessary number of batches to the Chernoff-bound:

$$Pr(k \geqslant \lceil n/2 \rceil + 1) = 1 - \sum_{k=1}^{L} C_n^k p^k q^{(n-k)} \geq 1, \tag{1}$$

where n is the number of batches, k is the number of correct outputs, $L = \lceil n/2 + 1 \rceil$, and $p = 1 - q$ is the probability of correct computation for each batch.

The reliability dependends on the number of batches and on the choice of the reliability-parameter (or confidence) r which is the probability to obtain a majority of wrong results. According to the Chernoff inequality, the required number of batches can be expressed as:

$$n \geqslant \frac{1}{(p-1)^2} \ln\left(\frac{1}{\sqrt{(1-r)}}\right) \geq 2, \tag{2}$$

For example, if the function computed is a quadratic polynomial then $p = (1-r)^3$ (as each batch must include at least three input points (vectors)). Equations (1) and (2) show that the use of majority-based choice among the results obtained from uniformly chosen batches can amplify the correction ability of a device. However, our previous results show that in spite of essential reduction in the number of batches needed for suitable computation accuracy, this number might be rather essential. The complexity (e.g., resource cost) of this approach depends on the number of queries, as well as on the decoding complexity of codes used for increasing correct computation probability for each of batches. In this talk we consider recent results in self-correcting computations. Possible ways to improve the amplification can minimize the number of queries (corresponding to the batch size) of the computation function performed to correct the computed value and accelerate the decrease in the error probability with the number of the batches used. Since we can interpret the error correcting as a "decoding of code-words" we can borrow ideas from *Locally Decodable Codes* (LDC). LDCs are error correcting codes where in order to retrieve the correct value of just one position of the input with high probability it is sufficient to read a small number of positions of the corresponding possibly corrupted codeword. The locally decodable code can recover from a much higher error-rate [4]. An example of LDC is the Hadamard code, which has the property that any input bit can be recovered with probability at least 1-2δ from δm code-words possibly corrupted in up to m positions, by a randomized algorithm that reads no more than 2 bits of the code in every invocation. One of the reasons for using LDC is that the previously used Reed-Solomon code consists of complete evaluations of polynomials of total degree up to d. In particular, there are LDCs which provide reduction of the error rate of the code with the number of queries which can be essentially higher d. That is the polynomial degree is not limited factor for the fraction of erroneous results reduction.

We will explore both the theory of Random self-reducibility and new results in Locally Decodable Codes for the problem of reconstruction of real numerical functions for correcting faults remaining in hardware after manufacturing testing.

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Consistent Sequences of Tests for Finite Markov Chains

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For each positive integer n we consider the statistical problem of testing a simple hypothesis $H_{0,\,n}$ against a complex alternative $H_{1,\,n}$. Each criterion is defined by a critical set S_n. S_n consists of all elementary events that lead to the acceptance of $H_{1,\,n}$.

In finite spaces it is important to consider complexity of an algorithm for calculation that data belongs to S_n. In previous studies [2], [3] we introduced a definition of a ban for a probability measure on a finite space. A ban means a sequence which has probability zero in a finite space. We have shown that the notion of bans is convenient because it allows to determine the critical set in the simplest way for calculation [2].

Let X be a finite set, X^n be a Cartesian product of X, X^∞ be a set of all sequences where i-th element belongs to X. Define $\mathcal{A}$ be a σ-algebra on X^∞, generated by cylindrical sets. $\mathcal{A}$ is also Borel σ-algebra in Tichonof product X^∞, where X has a discrete topology [1].

Then on $(X^\infty,\,\mathcal{A})$ a probability measure P_0 is defined. Assume $P_{0,\,n}$ be a project of P_0 on the first n coordinates of sequences from X^∞. It is clear that for every $B_n \subseteq X^n$

$$P_{0,\,n}(B_n) = P_0(B_n \times X^\infty).$$

Let $D_{0,\,n}$ be a support of measure $P_{0,\,n}$:

$$D_{0,\,n} = \left\{\vec{x}_n \in X^n,\, P_{0,\,n}(\vec{x}_n) > 0\right\}.$$

Denote $\Delta_{0,\,n} = D_{0,\,n} \times X^\infty$. The sequence $\Delta_{0,\,n}$, $n = 1, 2, ...$, is nonincreasing and

$$\Delta_0 = \lim_{n\to\infty} \Delta_{0,\,n} = \bigcap_{n=1}^{\infty} \Delta_{0,\,n}.$$

The set Δ_0 is closed and it is a support of P_0.

We also have a set of probability measures $\{P_\theta,\, \theta \in \Theta\}$ on $(X^\infty,\,\mathcal{A})$. Then as before we define $P_{\theta,\,n}$, $D_{\theta,\,n}$, $\Delta_{\theta,\,n}$, Δ_θ.

If $\overline{\omega}^{(k)} \in X^k$, then $\widetilde{\omega}^{(k-1)}$ is obtained from $\overline{\omega}^{(k)}$ by dropping the last coordinate.

Definition 1. Ban in measure $P_{0,\,n}$ is a vector $\overline{\omega}^{(k)} \in X^k$, $k \le n$, such that $P_{0,\,n}\left(\overline{\omega}^{(k)} \times X^{n-k}\right) = 0$. If $P_{0,\,k-1}(\widetilde{\omega}^{(k-1)}) > 0$ then $\overline{\omega}^{(k)}$ is the smallest ban.

If $\overline{\omega}^{(k)}$ is a ban in $P_{0,\,n}$ then for every $k \le s \le n$ and for every sequence $\overline{\omega}^{(s)}$ starting with $\overline{\omega}^{(k)}$ we have $P_{0,\,s}(\overline{\omega}^{(s)}) = 0$.

If there exists $\overline{\omega}^{(n)} \in X^n$ such that $P_{0,\,n}(\overline{\omega}^{(n)}) = 0$ then there exists the smallest ban.

By definition a critical set S_n of criterion is defined by bans if it includes all the extensions of the length n of some set of smallest bans. For the set S_n there exists a simple algorithm for computing the membership function for S_n. This algorithm calculates for each smallest ban its presence in the initial section of the vector, resulting in a statistical experiment.

Definition 2. Sequence of tests with critical sets S_n is called consistent (CST) [4] if $P_{0,\,n}(S_n) \longrightarrow 0$, $n \to \infty$, and $P_{\theta,\,n}(S_n) \longrightarrow 1$, $n \to \infty$, for every $\theta \in \Theta$.

Theorem 1[3]. There exists CST for which all critical sets are defined by bans iff $P_\theta(\Delta_0) = 0$ for every $\theta \in \Theta$.

Let all measures be finite homogeneous Markov chains which are defined by initial positive distributions $\vec{P_0}$, $\vec{P_\theta}$ and matrixes $\mathbf{P}_0 = \left\|P_{ij}^0\right\|$, $\mathbf{P}_\theta = \left\|P_{ij}^\theta\right\|$, We say that P_{ij}^θ contradicts P_{ij}^0 if $P_{ij}^0 = 0$ but $P_{ij}^\theta > 0$.

Theorem 2. There exists CST for which all critical sets are defined by bans iff for every $\theta \in \Theta$ and every ergodic class $\mathbf{P}_0$ there exists (ij) in it for whichP_{ij}^θ contradicts P_{ij}^0.

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Calculating the Probability Distribution for a Single-Link Model of the Triple Play Network

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Modern multi-service networks are inseparably linked with the commercial concept "triple play" [1] that implies simultaneous provisioning of telephony (VoIP, Skype, SIP-telephony), television (IPTV, VoD, streaming video via P2P) and data transmission (file transfer, e-mail, instant messaging) over a single broadband connection. Evidently, such various services generate traffics discriminating one from others not only in users' popularity and traffic volume but also in sensitivity to packet losses, bit-rate, duration, and etc. Three major traffic types are generally defined: unicast streaming, multicast streaming and elastic traffics. Streaming traffic is considered to be real-time and is characterized by a fixed duration, whereas elastic traffic is not real time and is assumed to have a variable duration and fixed volume. Unlike unicast traffic, multicast traffic has a network resources' saving nature achieved through employing multicast technology.

In terms of analysis of mathematical models with these three traffics, the teletraffic theory is developing step-by-step. Primarily, researchers proposed models allowing only for one of the traffics [2]. Then, they aimed to its pairwise combinations, i. e. unicast and multicast, unicast and elastic. For the models without joining heterogeneous streaming and elastic traffics, the analytical solutions and recursive algorithms were derived, while the mixture of unicast and elastic traffics requires approximate methods. For the first time, the model with three traffic types was proposed in [3], nevertheless, no exact algorithm was suggested there.

We consider a single link of C capacity units shared by unicast (u), multicast (m) and elastic (e) traffics. We assume all arrival rates λ_i to be Poisson and the resource occupancy durations to be exponential distributed with means μ_i^{-1}. The offered load is denoted by $\rho_i = \lambda_i \mu_i^{-1}$. Each type of traffic has a rate guarantee of b_i capacity units, $i \in \{\mathrm{u},\ \mathrm{m},\ \mathrm{e}\}$. The main distinction between three traffics is in service discipline: first come – first served (FCFS), the so-called "transparent" service [2], and egalitarian processor sharing (EPS) disciplines, respectively. It could be simply proofed that the process representing the system states is not a reversible Markov process and solution $p\,(n_{\mathrm{u}}, n_{\mathrm{m}}, n_{\mathrm{e}})$, $\sum_{i\in\{\mathrm{u},\ \mathrm{m},\ \mathrm{e}\}} b_i n_i \leqslant C$ of the equilibrium equations is not of product form.

Due to the limited space in the abstract, we result in the exact algorithm for calculating the stationary probability distribution for a model with two traffics – multicast and elastic. Its state space is given by $\mathcal{X}_1 = \Big\{(n_{\mathrm{m}}, n_{\mathrm{e}}) :\ 0 \leqslant \sum_{i\in\{\mathrm{m},\ \mathrm{e}\}} b_i n_i \leqslant C\Big\}$. So, the corresponding unnormalized probabilities can be computed as

$$q\,(0,0) = 1,$$

$$q\,(1,0) = \frac{\frac{\rho_{\mathrm{e}}}{C}\alpha^{0}_{0,\left\lfloor \frac{C}{b_{\mathrm{e}}}\right\rfloor -1} - \alpha^{0}_{0,\left\lfloor \frac{C}{b_{\mathrm{e}}}\right\rfloor}}{\alpha^{1}_{0,\left\lfloor \frac{C}{b_{\mathrm{e}}}\right\rfloor} - \frac{\rho_{\mathrm{e}}}{C}\alpha^{1}_{0,\left\lfloor \frac{C}{b_{\mathrm{e}}}\right\rfloor -1}},$$

$$q\,(n_{\mathrm{m}}, n_{\mathrm{e}}) = \alpha^{0}_{n_{\mathrm{m}},n_{\mathrm{e}}} + \alpha^{1}_{n_{\mathrm{m}},n_{\mathrm{e}}} \cdot q\,(1,0)\,, \quad (n_{\mathrm{m}}, n_{\mathrm{e}}) \in \mathcal{X}_1 \backslash \{(0,0)\,,\ (1,0)\}\,,$$

where coefficients $\alpha^{j}_{n_{\mathrm{m}},n_{\mathrm{e}}}$, $j \in \{0,\ 1\}$ are calculated by recursive formulas:

$$\alpha^{0}_{10} = 0, \quad \alpha^{1}_{10} = 1, \quad \alpha^{0}_{00} = 1, \quad \alpha^{1}_{00} = 0,$$

$$\alpha^{0}_{11} = -\frac{\lambda_{\mathrm{m}}}{(C - b_{\mathrm{m}})\,\mu_{\mathrm{e}}}, \quad \alpha^{1}_{11} = \frac{\lambda_{\mathrm{m}}\left(e^{\rho_{\mathrm{m}}} - 1\right)^{-1} + \lambda_{\mathrm{e}}}{(C - b_{\mathrm{m}})\,\mu_{\mathrm{e}}},$$

$$\alpha^{0}_{01} = \frac{\lambda_{\mathrm{m}} + \lambda_{\mathrm{e}}}{C\mu_{\mathrm{e}}}, \quad \alpha^{1}_{01} = -\frac{\lambda_{\mathrm{m}}\left(e^{\rho_{\mathrm{m}}} - 1\right)^{-1}}{C\mu_{\mathrm{e}}},$$

$$\alpha^{j}_{1,n_{\mathrm{e}}} = \left(1 + \alpha^{1}_{11}\right)\alpha^{j}_{1,n_{\mathrm{e}}-1} + \alpha^{0}_{11}\alpha^{j}_{0,n_{\mathrm{e}}-1} - \tfrac{\rho_{\mathrm{e}}}{C-b_{\mathrm{m}}}\alpha^{j}_{1,n_{\mathrm{e}}-2}, \quad n_{\mathrm{e}} = 2, \ldots, \left\lfloor \tfrac{C-b_{\mathrm{m}}}{b_{\mathrm{e}}}\right\rfloor,$$

$$\alpha^{j}_{0,n_{\mathrm{e}}} = \left(1 + \alpha^{0}_{01}\right)\alpha^{j}_{0,n_{\mathrm{e}}-1} + \alpha^{1}_{01}\alpha^{j}_{1,n_{\mathrm{e}}-1} - \tfrac{\rho_{\mathrm{e}}}{C}\alpha^{j}_{0,n_{\mathrm{e}}-2}, \quad n_{\mathrm{e}} = 2, \ldots, \left\lfloor \tfrac{C-b_{\mathrm{m}}}{b_{\mathrm{e}}}\right\rfloor + 1,$$

$$\alpha^{j}_{0,n_{\mathrm{e}}} = \alpha^{j}_{0,n_{\mathrm{e}}-1} + \tfrac{\rho_{\mathrm{e}}}{C}\left(\alpha^{j}_{0,n_{\mathrm{e}}-1} - \alpha^{j}_{0,n_{\mathrm{e}}-2}\right), \quad n_{\mathrm{e}} = \left\lfloor \tfrac{C-b_{\mathrm{m}}}{b_{\mathrm{e}}}\right\rfloor + 2, \ldots, \left\lfloor \tfrac{C}{b_{\mathrm{e}}}\right\rfloor.$$

It can be shown that the complexity of computation on the proposed algorithm is $O\left(|\mathcal{X}_1|\right)$ that the two times smaller than the complexity of equilibrium equations solution by the Gaussian elimination. For a system with three traffics a similar algorithm could not be obtained, but we derived an algorithm reducing the dimension of equilibrium equations to value $\left\lfloor \frac{C}{b_{\mathrm{u}}}\right\rfloor + \left\lfloor \frac{C-b_{\mathrm{m}}}{b_{\mathrm{u}}}\right\rfloor + 1$. For example, for initial data very close to reality, the dimension of the original system of equations is about 10^7, whereas the number of equations after the algorithm' application is only about 10^2.

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Markov Deision Process In The Model Of Distributed Computing System

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The substance of the report may be characterized as attempt to attract Markov Decision Process theory (MDP) to computing resource management. Thus, two different fundamental problems underlie the matter. One of them can be formulated as the necessity of further development of an adaptive variant of partially observed MDP. Another relates to the intensive computing systems development and consists in need of their effective management.

MDP is one of the most popular means commonly used for theoretical description in various areas concerning stochastic optimization. This occurs because of universal and expressive facilities of this language, as well of a good possibility to obtain strict mathematical assertions. However, it is not so serene with MDP applications in real systems requiring efficient and quick operating algorithms. Difficulties of the implementation have caused certain skepticism about MDP use advisability (see Howard [1]). The following reasons generate problems for MDP application: high state space dimensionality, incomplete state observation and missing of full transition matrix information. All three factors were in preceding years and are under steadfast attention of researchers at present. New directions were evolved from MDP theory, such as partially observed MDP and adaptive MDP. Many theoretical results and constructive algorithms that have found practical application have been obtained and contained in theoretical kernel of closely related branches (like reinforcement learning or artificial intelligence). The bibliography in the complete review of MDP theory is unbounded and the main sources are well known. The history of the subject is inseparably connected to works of Russian researchers that are practically unknown today. The most of the ideas and results in adaptive variant of partially observed MDP obtained up to 1990s (first of all in Russia but not only) are stated in Sragovich [2]. As to the modern state of the theory only one line of investigation which is most closely to this work will be referred here. Namely the case in the point is the gradient approach to the MDP developed mainly and owing to the author Cao [3]. But our results were obtained entirely independent from this works (see Konovalov [4]).

The extended intensive development of computer power does not eliminate the problem of its effective utilization but even aggravates it. One way of problem solving is creation of multiaccess distributed computing systems, the most essential example of which is grid. Some of these systems are remarkable for large number of participants (resource holders, consumers, brokers etc.) with own aims and own independent behavior. The problem considered in the wide sense has sufficiently long history and extensive related literature. The majority of tasks are concentrated in the framework of scheduling. Behavioral aspects are also intensive investigated recent years, particularly in view of grid problems. As to concrete review, one could be addressed, for example, to survey Xhafa and Abraham [5] that contains many references.

This report relates to the workflows planning problems, that are arising in such systems and that are common for almost all of them regardless of constructive and technical realization. The attention is focused on two matter of principal: how should be shared the available computing resources among consumers presented in the system, and what means should be applied to force participants to operate for the sake of hole system but not only in the interests of one's own. The selected method of attack is mathematical modeling and simulation. The basic idea lies in presence of the special modification of adaptive algorithms for partially observed multidimensional Markov decision process.

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Estimation of the overflow and loss probability in some Gaussian queues

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Gaussian processes are well-recognized models to describe the traffic dynamics of a wide class of modern telecommunication networks [3]. We consider the so-called fluid queue with a constant service rate and a Gaussian input process. The work focuses on the estimation of the overflow probability, that is the probability that the workload process exceed a threshold b (in the infinite buffer case) and the loss probability P_{loss}, or the buffer overflow probability (in the finite buffer queue). Such a probability is an important ingredient of the QoS analysis of telecommunication systems. At the same time, for the queues with general Gaussian input (in particular, for the most important models with fractional Brownian input (FBI)) there are no explicit results, and only some asymptotics for the overflow probability are found [1,2]. Thus, in general, only simulation remains an available and the most adequate way to estimate the required probability. It follows from the continuous time Lindley recursion, that in the infinite buffer queue, the problem is reduced to analysis of extremes of Gaussian processes.

Finite buffer systems, being more realistic models of real-life networks, are more difficult to be analyzed, and by this reason explicit (and asymptotic) expressions for P_{loss} in such systems are much less available.

In this work, we present a general formula connecting P_{loss} and the idle probability P_0. The estimate of probability P_0 is more available in the queues with large (finite) buffer under a light traffic, when a loss turns out to be a rare event. For the Brownian input (BI), we apply a regenerative methodology to construct confidence interval for P_{loss}.

Finally, some numerical results for queues with FBI and BI are presented.

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On the estimation of the overflow probability in finite buffer systems

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Finite buffer models play an important role in modeling the modern communication systems. In such system, the stationary loss probability is an important characteristic of the QoS provided by the system. Moreover, the output process is often an input to another node of a communication system.

In this work, we focus on the finite buffer systems possessing a regenerative property, Bratley and Fox [3], Glynn and Iglehart [4]. Regenerative structure of a wide class of the loss systems is described in detail. In particular, the embedded k-regenerations related to arrivals in $GI/M/m/n$ system, and to departures in $M/G/1/n$ system, are defined.

Moreover, we present both known analytical results for finite buffer systems and less known formula connecting stationary loss probability and stationary idle probability. The latter result is then used to evaluate stationary loss probability via estimation the idle probability. This approach is expected to be effective when a loss is a rare event, for instance, under large buffer and in the light traffic regime. Moreover, regenerative simulation is used for confidence estimation the loss probability.

The same methodology is also used to estimate the stationary blocking probability in a retrial bufferless systems with a constant retrial rate and with single or two servers. This research is based on the stability analysis presented in Avrachenkov and Morozov [1] and Avrachenkov, Goricheva and Morozov [2].

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Stability analysis of a multiprocessor model describing a high performance cluster

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In this work, we discuss a new multiprocessor queueing system describing dynamics of a high performance cluster with a large number m of processors serving a flow of tasks arriving to a single queue (with inter-arrival times T_i being a renewal process). A distinctive feature of the model is that upon the i-th arrival task requires a random number $1 \leqslant N_i \leqslant m$ of processors which must start to serve the new task simultaneously, with equal times S_i needed for service on each of N_i processors. It is assumed that $\{N_i\}$ is an i.i.d sequence.

It results in a delay caused a waiting of a task until all required number of processors become free. In turn, this implies that service discipline in this model is non work-conserving. While the problem of stability of classical multiprocessor queueing system is well-addressed (see Morozov [4 ,5], Asmussen [1]), stability analysis of this non work-conserving system is much more difficult than classical one.

The basic Markov workload process in this model is described by natural extension of the well-known Kiefer-Wolfowitz representation. Namely, denoting $W_i \in \mathsf{R}^m$ a workload vector in the system (which shows the amount of unfinished work on each of m servers, sorted in an ascending order), the recursive equation for W_i evaluation goes as follows

$$W_{i+1} = R \begin{pmatrix} W_i(N_i) + S_i - T_i \\ \cdots \\ W_i(N_i) + S_i - T_i \\ W_i(N_i+1) - T_i \\ \cdots \\ W_i(m) - T_i \end{pmatrix}^{+} .$$

Here operator R puts the components of a vector in ascending order, and $(\cdot)^+ = \max(0, \cdot)$ is taken componentwise. We note that N_i components of a vector are identical.

As a result, stability analysis in this contribution is treated in the terms of positive Harris recurrence of the basic process (cf. Thorrison [7]). However, in general setting exact stability analysis of such a model implying tight stability region seems not tractable.

Instead we construct some minorant and majorant classical systems, which allow to obtain a tight stability region in several important cases, including a large number m of processors. Some specific distributions of N (and connection between N and m) are also considered.

Then we develop regenerative stability analysis of these simpler systems provided classical regeneration exists. For a more general model, we construct the so-called one-dependent regeneration structure which indeed is equivalent to Harris recurrence of the underlying Markov process.

Some numerical results related to stability analysis are presented.

An important ingredient of the work is also a survey of main moment properties of the workload process for a wide class of queueing systems (see Scheller-Wolf, Vesilo [6]).

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The use of Chebyshev and Gegenbauer polynomials in the analysis of finite queue with negative customers and bunker for ousted customers

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At present queueing systems (QS) and queueing networks with negative customers remain the subject of extensive research. As the evidence one can cite many works in this field of study that are being printed in periodicals each year. For the latest bibliography on the subject see Tien Van Do [1]. Classical negative customer, as opposed to regular, upon entering the system either removes a regular customer from the system, if there is one, or converts it into some other customer.

In the present work (as well as in Manzo, Cascone and Razumchik [2], Pechinkin and Razumchik [3]) a slightly different type of negative customers is considered. Upon entering the system a negative customer does not remove a regular customer from the system, but displaces it into another queue. So the displaced customer stays the same except for the fact that it is served according to a certain discipline. QS with such negative customers can model, for example, fault-related processes in distributed computing system and in databases with two-phase commit strategy.

Consider a single-line queueing system with finite buffer of size r, incoming Poisson flow of regular customers of intensity λ and Poisson flow of negative customers of intensity λ^-. A negative customer upon arrival pushes a regular customer out of the queue in buffer (if it is not empty) and moves it to the queue of finite capacity r in bunker. If upon arrival of a negative customer the queue in the buffer is empty, it leaves the system having no effect on it. Customers from both queues are served according to exponential distribution with parameter μ, FCFS discipline, but customers in bunker are served with a relative priority (i.e. have lower priority than customers in buffer). At last, if upon arrival of a regular customer the buffer is full, this claim is lost; if upon arrival of a negative customer the buffer is not empty and bunker if full, displaced customer from buffer is lost.

The goal is to find the stationary probabilities of number of customers in buffer and bunker. The functioning of the system can be described by homogeneous Markov process $X(t) = \{\xi(t), \eta(t)\}$ (where $\xi(t)$ – number of customers in buffer, $\eta(t)$ – number of customers in bunker) with the three-diagonal block matrix of transition intensities Q of size $((r+1)^2+1) \times ((r+1)^2+1)$. So, in order to find stationary probabilities, one may use matrix-geometric method for generalized birth-and-death processes (see, for example, Latouche and Ramaswami [4]) or elimination method (see, for example, Bocharov, D'Apice, Pechinkin and Salerno [5]). But it was found that there exists another one way to obtain stationary probabilities for this QS. This way is not new as it was already used in Avrachenkov, Vilchevsky and Shevljakov [6], but for the analysis of another type of QS.

It can be shortly described as follows. Let us denote by p_{ij} the stationary probability of i customers in buffer and j customers in bunker, and let p_0 be the stationary probability of the empty system. At first one should find two-dimensional PGF $P(u,v) = \sum_{i=0}^{r}\sum_{j=0}^{r} p_{ij}u^i v^j$. Then using properties of PGF and the form of marginal distributions one obtains two equations that can be solved using series expansion with the help of Chebyshev and Gegenbauer polynomials. The solution represents the expressions for the probabilities p_{ir}, p_{0i} and p_{rj} that depend only on λ, λ^-, μ and p_0. Then, using local balance principle and normalizing condition one can find the expression for p_0 and then compute all p_{ij}.

This method shows good speed and accuracy during computational experiments compared to two methods, mentioned above. Moreover, matrix-geometric and elimination methods need to be adapted to the structure of matrix Q. Otherwise, in case of large values of r, they work much slower than the method, based on special functions.

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Software Tools for Circular Stochastic Systems Analysis

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1. Modern analytical modeling stochastic information technologies for nonlinear stochastic systems (StS) in Euclidean spaces based on quasilinear methods for solving equations (Eqs) for probability densities. Corresponding methods and instrumental software are described in [1, 2]. Methods and software tools for problems of circular mathematical statistics are observed in [3]. The paper is devoted to instrumental software tools for circular StS (CStS) based on equivalent statistical linearization [4].

2. Following [4] let us consider scalar circular random variable $Y = \varphi(X)$ being nonlinear deterministic function of scalar circular random variable (CRV) X. Using mean square circular criteria for characteristic CRV functions Y and U we approximate $\varphi(X)$ by the following equivalent linear CRV $U = k_0\mu + k_1(X-\mu)$, μ being circular mean for X; k_0 and k_1 being statistical linearization coefficients depending on equivalent probability density.

For "wrapped" scalar normal density (WN) with parameters μ and σ we have complex Eq $M_{WN}\exp[i\varphi(X)] = M_{WN}\exp\big[ik_0(\mu,\sigma)\mu + k_1(\mu,\sigma)(X-\mu)\big]$.

For 3 typical nonlinear functions $Y = \varphi(X)$ we get the following expressions for k_0 and k_1:

- Example 1.

$$y = l\,sgn(x), \qquad k_0 = \frac{1}{\mu}\arctan\left(\frac{2\sin l\Phi(\frac{\mu}{\sigma})}{\cos l}\right),$$

$$k_1 = \frac{1}{\sigma}\sqrt{-2\ln\sqrt{\cos^2 l + 4\sin^2 l\left(\Phi(\frac{a}{\sigma})\right)^2}}.$$

- Example 2.

$$y = l\,1(x), \qquad k_0 = \frac{1}{\mu}\arctan\left(\frac{\sin l\ \left(\frac{1}{2}+\Phi\left(\frac{\mu}{\sigma}\right)\right)}{\frac{1}{2}-\Phi\left(\frac{\mu}{\sigma}\right)+\cos l\ \left(\frac{1}{2}+\Phi\left(\frac{\mu}{\sigma}\right)\right)}\right),$$

$$k_1 = \frac{1}{\sigma}\sqrt{-2\ln\sqrt{\frac{1}{2}(1+\cos l) + 2(1-\cos l)\Phi\left(\frac{\mu}{\sigma}\right)^2}}.$$

- Example 3.

$$y = l\,x\,1(x), \qquad k_0 = \frac{1}{\mu}\arctan\left(\frac{\frac{1}{2}e^{-\frac{l^2\sigma^2}{2}}\sin l\mu}{\frac{1}{2}-\Phi\left(\frac{\mu}{\sigma}\right)+\frac{1}{2}e^{-\frac{l^2\sigma^2}{2}}\cos l\mu}\right),$$

$$k_1 = \frac{1}{\sigma}\sqrt{-2\ln\sqrt{\left(\frac{1}{2}-\Phi\left(\frac{\mu}{\sigma}\right)\right)^2+e^{-\frac{l^2\sigma^2}{2}}\cos l\mu\left(\frac{1}{2}-\Phi\left(\frac{\mu}{\sigma}\right)\right)+\frac{1}{4}e^{-\frac{l^2\sigma^2}{2}}}},$$

in the specific case when $\mu = a = 0$

$$k_1 = \frac{1}{\sigma}\sqrt{-2\ln\sqrt{\frac{1}{4}+\frac{3}{4}e^{-\frac{l^2\sigma^2}{2}}}}.$$

Using linearized Eqs for CStS we get deterministic Eqs [4] for mathematical expectations, variances and covariances for times t and t' ($t' > t$).

The original software tools "CStS-analysis" is instrumented in MATLAB for nonlinear discrete and continuous CStS. The current experimental version of "CStS-analysis" uses functions of MATLAB Symbolic Math toolbox and presents the set of open program functions with numerical and graphic output. Applications: precise circular mechanisms, statistical dynamics of the Earth motion, etc [4, 5].

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On a queue length in queueing system with preemptive loss priority

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This paper is concerned with one-channel preemptive loss priority queues in which customers arrive according to a renewal process with hyperexponential distribution. Entering customers are separated into two priority classes with probability p_1 and p_2. Customers of the first class have priority to customers of the second class. Service times are jointly independent random variables with distribution function $B_i(x)$, density $b_i(x)$ and Laplace-Stieltjes transform $\beta_i(s)$ for customers of $i-$th class.

Let $a(x) = \sum\limits_{j=1}^{N} c_j a_j \exp\left(-a_j x\right), x \geqslant 0,\ a_i \neq a_j,\ i \neq j,\ c_j > 0,\ \sum\limits_{i=1}^{N} c_i = 1,$ be the density of interarrival time. We set: $L_i(t)$ – the number of customers of $i-$th class, $i = 1, 2$, in the system at the time instant t, $p(z_1, z_2, s) = \int\limits_0^\infty e^{-st}\mathbf{E} z_1^{L_1(t)} z_2^{L_2(t)} dt$, $\mu_1(z_1, z_2), \ldots, \mu_N(z_1, z_2)$ be continuous solutions of the equation $\prod\limits_{i=1}^{N}(\mu + a_i) = (p_1 z_1 + p_2 z_2)\sum\limits_{j=1}^{N} c_j a_j \prod\limits_{i\neq j}(\mu + a_i)$, $\alpha_k(z_1, z_2) = \prod\limits_{i\neq k}(\mu_k(z_1, z_2) - \mu_i(z_1, z_2))$,

Lemma 1. *For each $k = 1, \ldots, N$ the functional equation $z_1 = \beta_1\left(s - \mu_k(z_1, z_2)\right)$ has a unique solution $z_1 = z_1^{(k)}(z_2, s)$, which is analytic in the region $|z_2| < 1$, $\operatorname{Re} s > 0$.*

Let $e_k(z_2, s) = \sum\limits_{l=1}^{N}\left(p_2 z_2 - \frac{\prod\limits_{j=1}^{N}(\psi_j(z_2,s)+a_l)}{c_l a_l \prod\limits_{i\neq l}(a_l - a_i)}\right)\prod\limits_{j\neq l}^{N}(\mu_k(0, z_2) + a_j) c_l a_l.$

Lemma 2. *The functional equation*

$$\prod_{l=1}^{N}\left(1 - z_2^{-1}\beta_2(s - \mu_l(0, z_2))\right) + \sum_{j=1}^{N}\prod_{l\neq j}\left(1 - z_2^{-1}\beta_2(s - \mu_l(0, z_2))\right) \times$$
$$\times \frac{1 - \beta_2(s - \mu_j(0, z_2))}{s - \mu_j(0, z_2)} \cdot \frac{e_j(z_2, s)}{\alpha_j(0, z_2)} = 0$$

has N solutions $\zeta_1(s), \ldots, \zeta_N(s)$ which are analytic in the region $\operatorname{Re} s > 0$.

We set: $\psi_k(z_2, s) = \mu_k\left(z_1^{(k)}(z_2, s), z_2, s\right), \psi_{mk}(s) = \psi_m(\zeta_k(s), s), \mu_{jk}(s) = \mu_j(0, \zeta_k(s)), \omega_{kl}(s) = \sum\limits_{j=1}^{N}\frac{1-\beta_2(s-\mu_{jk}(s))}{s-\mu_{jk}(s)}\cdot\frac{\prod\limits_{f\neq l}(\mu_{jk}(s)+a_f)}{\alpha_j(0,\zeta_k(s))(1-\zeta_k^{-1}(s)\beta_2(s-\mu_{jk}(s)))}.$

Theorem 1. *A function $p(z_1, z_2, s)$ is determined by the formula*

$$p(z_1, z_2, s) = \sum_{j=1}^{N} \left(p_{0j}(s) + c_j \sum_{k=1}^{N} \left(\frac{1 - \beta_1(s - \mu_k(z_1, z_2))}{s - \mu_k(z_1, z_2)} \times \right.\right.$$

$$\left.\left. \times \frac{\gamma_1^{(k)}(z_1, z_2, s)}{\mu_k(z_1, z_2) + a_j} + \frac{1 - \beta_2(s - \mu_k(0, z_2))}{s - \mu_k(0, z_2)} \frac{\gamma_2^{(k)}(z_2, s)}{\mu_k(0, z_2) + a_j} \right)\right),$$

where

$$\frac{\left(1 - z_1^{-1}\beta_1(s - \mu_k(z_1, z_2))\right) \alpha_k(z_1, z_2)}{p_1 z_1 + p_2 z_2} \gamma_1^{(k)}(z_1, z_2, s) =$$

$$= \prod_{j=1}^{N} (\mu_k(z_1, z_2) - \psi_j(z_2, s)) \cdot q^{(1)}(z_2, s),$$

$$q^{(1)}(z_2, s) = \sum_{\nu=1}^{N} a_\nu \left(p_{0\nu}(s) + c_\nu \sum_{k=1}^{N} \frac{\gamma_2^{(k)}(z_2, s)}{\mu_k(0, z_2) + a_\nu} \cdot \frac{1 - \beta_2(s - \mu_k(0, z_2))}{s - \mu_k(0, z_2)} \right),$$

$$\left(\prod_{l=1}^{N} \left(1 - z_2^{-1}\beta_2(s - \mu_l(0, z_2))\right) + \sum_{j=1}^{N} \prod_{l \neq j} \left(1 - z_2^{-1}\beta_2(s - \mu_l(0, z_2))\right) \times \right.$$

$$\left. \times \frac{1 - \beta_2(s - \mu_j(0, z_2))}{s - \mu_j(0, z_2)} \cdot \frac{e_j(z_2, s)}{\alpha_j(0, z_2)} \right) \cdot \alpha_k(0, z_2) \cdot \gamma_2^{(k)}(z_2, s) =$$

$$= \prod_{l \neq k} \left(1 - z_2^{-1}\beta_2(s - \mu_l(0, z_2))\right) \left(r_k(z_2, s) - \sum_{j \neq k} \frac{1 - \beta_2(s - \mu_j(0, z_2))}{s - \mu_j(0, z_2)} \cdot \right.$$

$$\left. \cdot \frac{e_k(z_2, s) \cdot r_j(z_2, s) - e_j(z_2, s) \cdot r_k(z_2, s)}{1 - z_2^{-1}\beta_2(s - \mu_j(0, z_2))} \cdot \frac{1}{\alpha_j(0, z_2)} \right).$$

and functions $p_{0j}(s)$ are determined from the systems of linear equations

$$\sum_{l=1}^{N} \left((s + a_l) \cdot \omega_{kl}(s) - \sum_{\nu=1}^{N} \omega_{k\nu}(s) \cdot \frac{\prod\limits_{m=1}^{N} (\psi_{mk}(s) + a_\nu)}{\prod\limits_{i \neq \nu} (a_\nu - a_i)} \right) \times$$

$$\times a_l p_{0l}(s) = \sum_{l=1}^{N} c_l a_l \omega_{kl}(s), \ k = 1, \ldots, N.$$

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The Analysis of RED-Like Algorithms Characteristics Based on Queueing Systems with Batch Arrival

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The system description. The queueing system $GI^B/M/1/R_{q_1,q_2}$ with finite queue size R ($R < \infty$) (in packets) and two thresholds q_1 q_2, $0 \leqslant q_1 < q_2 \leqslant R$, is discussed. The packets arrive in batches of size l, $l = \overline{1, L}$, $L < \infty$ (batch size — in packets).

The probability of arrival of the batch of l packets is a_l, $\sum\limits_{l=1}^{L} a_l = 1$.

The service time distribution is exponential with parameter μ, arrived packets are served by one.

This work is the continuation of research presented in [1, 2]. The main difference from papers [1, 2] is that we consider the queueing system with batch arrival (so we need to consider batch length probability distribution, system overflow due batch arrival, introduced thresholds) and don't use general renovation [3–5].

Let's q be the number of packets in our system (with a packet on a server) and there is an arriving batch of l ($l = \overline{1, L}$) packets, then:

- if $q + l > R + 1$, then the all arriving batch is dropped;
- if $q + l \leqslant R + 1$, then:
  - $0 \leqslant q \leqslant q_1$ — the whole batch is accepted;
  - $q_1 < q \leqslant q_2$ — either the arriving batch is accepted with probability $1 - p(q)$, or it is dropped (the whole batch) with probability $p(q)$;
  - $q_2 < q \leqslant R + 1$ — the whole batch is dropped with probability equal to one.

There are different possibilities of queue size R and maximal batch length L interaction: 1) $R < L < \infty$, 2) $q_2 < L \leqslant R$, 3) $q_1 < L \leqslant q_2$, 4) $1 \leqslant L \leqslant q_1$.

The analitycal expressions for following probabilistic-time characteristics: the embedded Markov chain distribution of number of packets in the system, the probability that the arriving packet will be served, mean waiting time, are obtained. The results of this article generalize the results of [1, 2].

Probabilistic-time characteristics. The system is investigated with the help of embedded by moments of batch arrival Markov chain [3–5]. The steady-state probability distribution p_k^+ ($k = \overline{1, R+1}$) may be found by the following system solution:

$$p_k^+ = \sum_{j=1}^{R+1} p_j^+ \left(\sum_{i=0}^{\min(j,k-1)} A_{j,i} a_{k-i} \right) + \sum_{j=k}^{R+1} p_j^+ \dot{a}_k A_{j,k}, \ 1 \leqslant k \leqslant q_1, \qquad (1)$$

$$p_k^+ = \sum_{j=1}^{R+1} p_j^+ \left(\sum_{i=0}^{\min(j,q_1)} A_{j,i} a_{k-i} + \sum_{i=q_1+1}^{\min(j,k-1)} A_{j,i} a_{k-i}(1-p(i)) \right) + \\ + \sum_{j=k}^{R+1} p_j^+ A_{j,k}(\dot{a}_k + \tilde{a}_k), \quad q_1+1 \leqslant k \leqslant q_2, \quad (2)$$

$$p_k^+ = \sum_{j=1}^{R+1} p_j^+ \left(\sum_{i=0}^{\min(j,q_1)} A_{j,i} a_{k-i} + \sum_{i=q_1+1}^{\min(j,q_2)} A_{j,i} a_{k-i}(1-p(i)) \right) + \\ + \sum_{j=k}^{R+1} p_j^+ A_{j,k}, \quad q_2+1 \leqslant k \leqslant R+1, \quad (3)$$

with condition $\sum\limits_{k=1}^{R+1} p_k^+ = 1$. Here $A_{i,j} = \frac{(-\mu)^{(i-j)}}{(i-j)!} \alpha^{(i-j)}(\mu)$, $i = \overline{1, R+1}$, $j = \overline{0, i}$, $\alpha(s)$ — Laplace-Stieltjes transformation of probability distribution function of input flow; $\dot{a} = \sum\limits_{l=R+2-k}^{L} a_l$, $k = \overline{0, R+1}$; $\tilde{a}_k = p(k)(1 - \dot{a}_k)$, $k = \overline{0, R}$.

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